NEW SCHEME

CSE

CS34

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Third Semester B.E. Degree Examination, July/August 2005

Computer Science /Information Science and Engineering

Discrete Mathematical Structures

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions.*

1. (a) Determine the sets A and B, given that

$A - B = \{1,3,7,11\}$, $B - A = \{2,6,8\}$ and $A \cap B = \{4,9\}$ (4 Marks)

(b) prove that :

$A \Delta B = (B \cap \overline{A}) \cup (A \cap \overline{B}) = (B - A) \cup (A - B)$ (6 Marks)

(c) A survey of 500 televisions viewers of sports channel produced the following information : 285 watch cricket, 195 watch hockey, 115 watch foot ball, 45 watch cricket and foot ball, 70 watch cricket and Hockey, 50 watch Hockey and foot ball and 50 do not watch any of the three kinds of games.

i) How many viewers in the survey watch all three kinds of games?

ii) How many viewers watch exactly one the sports. (6 Marks)

(d) By mathematical induction, prove that $\lfloor\underline{n} \geq 2^{n-1}$ for all integers $n \geq 1$. (4 Marks)

2. (a) Define tautology and contradiction of a compound proposition. Prove that, for any propositions p, q, r the compound proposition :

$[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology. (8 Marks)

(b) Prove that $[(\neg p \vee q) \wedge (p \wedge (p \wedge q))] \Leftrightarrow p \wedge q$

Hence deduce that $[(\neg p \wedge q) \vee (p \vee (p \vee q))] \Leftrightarrow p \vee q$ (6 Marks)

(c) Define the following :

Rule of syllogism ii) Modus pones iii) Modus tollens.

Test whether the following argument is valid : If I drive to work, then I will arrive tired. I am not tired (when I arrive at work) Hence, I donot drive to work. (6 Marks)

3. (a) Define : i) Open sentences ii) Quantifiers.

Write down the following proposition in symbolic form, and find is negation :

"If all triangles are right-angled, then no triangle is equiangular". (7 Marks)

(b) Find whether the following is a valid argument for which the universe is the set of all students.

No engineering student is bad in studies.
Ram is not bad in studies.
Therefore, Ram is an engineering student. (7 Marks)

i) A direct proof

ii) An indirect proof and

iii) Proof by contradiction, for the following statement :
"If n is an odd integer, then $(n+9)$ is an even integer". **(6 Marks)**

4. (a) Let $A = \{1,2,3\}$ and $B = \{1,2,3,4\}$. The relations R and S from A to B are represented by the following matrices. Determine the relations $\overline{R}$, $R \cup S$ and $R \cap S$ and their matrix representations.

$$M(R) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \quad M(S) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix},$$ **(7 Marks)**

(b) Let $A = \{1,2,3,4\}$ and R a relation on A defined by
$R = \{(1,2), (1,3), (2,4), (3,2), (3,3), (3,4)\}$. Find R^2 and R^3. Write down the graphs of R, R^2 and R^3. **(6 Marks)**

(c) Define the following with one example each ;

i) Reflexive relation

ii) Symmetric relation

iii) Antisymmetric relation.

let R and S be relations on a set A. If R and S are symmetric, prove that $R \cap S$ also is symmetric. **(7 Marks)**

5. (a) Define an equivalence relation with an example. Let $A = \{1,2,3,4,5\}$. Define a relation R and $A \times A$ by $(x_1, y_1)R(x_2, y_2)$ if $x_1 + y_1 = x_2 + x_2$. Verify that R is an equivalence relation on $A \times A$. **(6 Marks)**

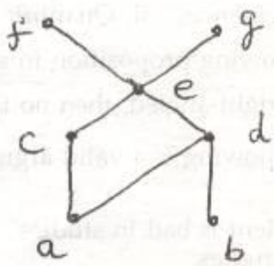
(b) Define a partial order on a set A with an example. let $A = \{1,2,3,4,6,12\}$. On A, define the relation R by aRb if a divides b. Prove that R is a partial order on A. Draw the Hasse diagram for this relation. **(7 Marks)**

(c) Let A be a set and $B \subseteq A$. Define

i) Least upper bound $(L \cup B)$ of B.

ii) Greatest lower bound (GLB) of B.

Consider the poset whose Hasse diagram is shown below. Find $L \cup B$ and GLB of the set $B = \{c, d, e\}$. **(7 Marks)**



6. (a) Define a function from a set A to the set B. Distinguish between a relation and a function. Let A and B be finite sets with $|A| = m$ and $|B| = n$. Find how many functions are possible from A to B? If there are 2187 functions from A to B and $|B| = 3$, what is $|A|$? (8 Marks)

(b) Define stirling number of the second kind. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{w, x, y, z\}$. Find the number of onto functions from A to B. (6 Marks)

(c) Define :

Permutation function ii) Characteristic function.

Given $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$, compute p^{-1} and p^{-2}. (6 Marks)

7. (a) Define an Abelian group. Let $(G, *)$ be the set of all non-zero real numbers and let $a * b = \frac{1}{2}ab$. Show that $(G, *)$ is an Abelian group. (8 Marks)

(b) Define a subgroup. Let G be a group and $G_1 = \{x \in G | xy = yx \text{ for all } y \in G\}$. Prove that G_1 is a subgroup of G. (6 Marks)

(c) Define a cyclic group with an example. Prove that every cyclic group is abelian. (6 Marks)

8. (a) State and prove Lagrange's theorem. (6 Marks)

(b) Define homomorphism and isomorphism in a group. let f be a homomorphism from a group G_1 to a group G_2. Prove that

i) If e_1 is the identify in G_1 and e_2 is the identity in G_2, then $f(e_1) = e_2$.

ii) $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G_1$. (8 Marks)

(c) The generator matrix for an encoding function $E : Z_2^3 \rightarrow Z_6^6$ is given by

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Find the code words assigned to 110 and 010. Also obtain the associated parity check matrix. (6 Marks)

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