NEW SCHEME

CS34

Reg. No. [ ]

Third Semester B.E. Degree Examination, January/February 2006

Computer Science/Information Science and Engineering

Discrete Mathematical Structures

Time: 3 hrs.) (Max.Marks : 100

Note: *1. Answer any FIVE full questions.*
2. All questions carry equal marks.

1. (a) Determine the sets A and B, given that $A - B = \{1, 2, 4\}$, $B - A = \{7, 8\}$, and $A \cup B = \{1, 2, 4, 5, 7, 8, 9\}$ **(4 Marks)**

(b) Prove that : $\overline{A \triangle B} = \overline{A} \triangle B = A \triangle \overline{B}$ **(6 Marks)**

(c) The freshman class of a private engineering college has 300 students. It is known that 180 can program in PASCAL, 120 in FORTRAN, 30 in C^{++}, 12 in PASCAL and C^{++}, 18 in FORTRAN and C^{++}, 12 in PASCAL and FORTRAN, and 6 in all three languages. If two students are selected at random, what is the probability that they can i) both program in PASCAL ? ii) both program only in Pascal. **(6 Marks)**

(d) Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(4 Marks)**

2. (a) If statement q has the truth value 1, determine all truth value assignments for the primitive statements p, r and s for which the truth vaue of the statement.

$$[q \rightarrow \{\neg p \vee r) \wedge \neg s\}] \wedge \{\neg s \rightarrow (\neg r \wedge q)\}$$ **(4 Marks)**

(b) Define tautology. Prove that, for any propositions p, q, r, the compound proposition $[(p \rightarrow q) \wedge (q \rightarrow r)]$ is a tautology. **(6 Marks)**

(c) Prove the following logical equivalences without using truth tables :

i) $p \vee [p \wedge (p \vee q)] \Longleftrightarrow p$

ii) $\neg [\neg \{p \vee q) \wedge r\} \vee \neg q] \Longleftrightarrow q \wedge r$ **(6 Marks)**

(d) Test whether the following argument is valid :

If interest rates fall, then the stock market will rise. The stock market will not rise. Therefore the interest rates will not fall. **(4 Marks)**

3. (a) Consider the following open statements with the set of all real numbers as the universe

$p(x) : x \geq 0, \; q(x) : x^2 \geq 0$

$r(x) : x^2 - 3x - 4 = 0, \; s(x) : x^2 - 3 > 0,$

then find the truth values of the following statement.

i) $\exists x [p(x) \wedge r(x)]$

ii) $\forall x [p(x) \rightarrow q(x)]$

iii) $\forall x [q(x) \rightarrow s(x)]$ **(6 Marks)**

Contd.... 2

(b) Write down the following propositions in symbolic form, and find its negation.

i) For all integers n, if n is not divisible by 2, then n is odd.

ii) All integers are rational numbers and some rational numbers are not integers **(7 Marks)**

(c) For the universe of all people, find whether the following is a valid argument :

All mathematics professors have studied calculus
Ramanujan is a mathematics professor
Therefore Ramanujan has studied calculus. **(7 Marks)**

4. (a) Define cartesian product of two sets. For any three non-empty sets A, B, C prove that $A \times (B - C) = (A \times B) - (A \times C)$ **(6 Marks)**

(b) Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by aRb if and only if a is c multiple of b. Write down the relation matrix $M(R)$ and draw its digraph. **(6 Marks)**

(c) If $A = \{1, 2, 3, 4\}$ and R, S are relations on A defined by $R = \{(1,2), (1,3), (2,4), (4,4)\}$, $S = \{(1,1)\ (1,2)\ (1,3)(1,4), (2,3)\ (2,4)\}$ find RoS, SoR, R^2 and S^2. **(4 Marks)**

(d) Let $A = \{a, b, c, d, e\}$. Consider the partition $P = \{\{a, b\}, \{c, d\}\{e\}\}$ of A. Find the equivalance relation inducing this partition. **(4 Marks)**

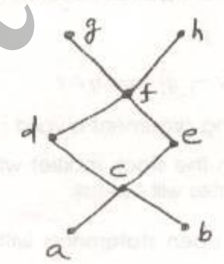
5. (a) Let $A = \{1, 2, 3, 4, 5\}$. Define a relation R on $A \times A$ by $(x_1, y_1)\ R(x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.

i) Verify that R is an equivalence relation on $A \times A$

ii) Determine the equivalence classes $[(1,3)]$, $[(2,4)\ and\ [1,1]]$ **(6 Marks)**

(b) Define a partially ordered set. If R is a relation on the set $A = \{1, 2, 3, 4\}$ defined by xRy if x divides y, prove that (A, R) is a poset. Draw its Hasse diagram. **(7 Marks)**

(c) Let A be a set and $B \subseteq A$. Define i) Least upper bound of B ii) Greatest lower bound of B. Consider the poset whose Hasse diagram is shown below Find LUB and GLB of the set $B = \{c, d, e\}$



(7 Marks)

6. (a) Prove that, a function $f : A \longrightarrow B$ is invertible if and only if it is one-one and onto. **(6 Marks)**

(b) Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of the set $A = \{1, 2, 3, 4, 5, 6\}$.

i) Write p as a product of disjoint cycles

ii) Compute p^{-1}

iii) Compute p^2 and p^3

iv) Find the smallest integer k such that $p^k = I_A$ **(7 Marks)**

Define characteristic funtion. For any sets A, B contained in a universal set U, prove that:

i) $f_{\overline{A}}(x) = 1 - f_A(x)$

ii) $f_{A \cap B}(x) = f_A(x) \cdot f_B(x)$ **(7 Marks)**

Define an abelian group. Prove that a group G is abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$. **(6 Marks)**

Prove that, if H is a non empty subset of a group G, then H is a subgroup of G if and only if

i) for all $a, b \in H,\ ab \in H$ and

ii) for all $a \in H,\ a^{-1} \in H$ **(6 Marks)**

Define left and right cosets. State and prove Lagrange's theorem. **(8 Marks)**

Define a ring. If R is a ring with unity and $a,\ b$ are units in R, prove that ab is a unit in R and that $(ab)^{-1} = b^{-1}a^{-1}$. **(6 Marks)**

If $f : G \longrightarrow H,\ \ g : H \longrightarrow K$ are homomorphisms, prove that the composite function $gof : G \longrightarrow K$, where $(gof)\ (x) = g(f(x))$, is a homomorphism. **(6 Marks)**

Define a group code. Let $E : {Z_2}^m \longrightarrow {Z_2}^n$ be an encoding function given by a generator matrix G or the associated parity - check matrix H. Then prove that $C = E({Z_2}^m)$ is a group code. **(8 Marks)**

** * **