CS34

USN

NEW SCHEME

Third Semester B.E. Degree Examination, Dec.06 / Jan.07
CS / IS
Discrete Mathematical Structures

Time: 3 hrs.] [Max. Marks:100

Note: ***1. Answer any FIVE full questions.***

1 a. If S, T $\subseteq$ U, prove that S and T are disjoint if and only if $S \cup T = S \Delta T$. **(04 Marks)**

b. Simplify the expression $\overline{(A \cup B \cap C) \cup \overline{B}}$. **(04 Marks)**

c. A problem is given to four students A, B, C, D whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ respectively. Find the probability that the problem is solved. **(04 Marks)**

d. i) Prove by mathematical induction that for every positive integer n, 3 divides $n^3 - n$.

ii) Find the explicit formula for $c_n = c_{n-1} + n$, $c_1 = 5$. **(08 Marks)**

2 a. Let p and q be primitive statements for which $p \to q$ is false. Determine the truth values of the following :

i) $p \wedge q$ ii) $\sim p \vee q$ iii) $q \to p$ iv) $\sim q \to \sim p$ **(05 Marks)**

b. Prove the following logical equivalence without using truth table :

$(p \to q) \wedge [\sim q \wedge (r \vee \sim q)] \Leftrightarrow \sim (q \vee p)$. **(05 Marks)**

c. Prove that for all integer k and l, if k and l are both odd then k + l is even and kl is odd. **(05 Marks)**

d. Find whether the following argument is valid :

No engineering student of first or second semester-studies logic.

Anil is an engineering student who studies logic

Therefore Anil is not in second semester. **(05 Marks)**

3 a. Define partition set. List all partitions of P = {1, 2, 3} **(04 Marks)**

b. Let A = {a, b, c, d} and R = {(ab), (bb), (cb), (cd), (da), (ac)}

Compute i) R^2 ii) R^∞ iii) $M_R{}^2$ iv) $M_R{}^6$ **(04 Marks)**

c. Let A = {1, 2, 3, 4, 6, 12} on A. Define the relation R by aRb if and only if 'a' divides 'b'. Prove that R is a partial order on A. Draw the Hass diagram for this relation. **(04 Marks)**

d. Let A = {1, 2, 3, 4, 5} define a relation R on A × A by $(x_1, y_1) R (x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.

i) Verify that R is an equivalence on A × A.

ii) Determine the equivalence classes $[(1,3)]$, $[(2,4)]$ and $[(1,1)]$.

iii) Determine the partition of A × A induced by R. **(08 Marks)**

4 a. In each of the following cases sets A and B and a function 'f' from A to B are given. Determine whether f is one to one or not or both or neither.

i) A = {1, 2, 3, 4} B = {1, 2, 3, 4} f = {(1 1), (2 3), (3 4), (4 2)}

ii) A = {a, b, c} B = {1, 2, 3, 4} f = {(a 1), (b 1), (c 3)}

iii) A = {1, 2, 3} B = {1, 2, 3, 4, 5} f = {(1 1), (2 3), (3 4)}

iv) A = {1, 2, 3} B = {1, 2, 3, 4, 5} f = {(1 1), (2 3), (3 3)} **(06 Marks)**

b. Let f : A → B and g : B → C be any two functions. Then the following are true.

i) If f and g are one-to-one so is $g \circ f$.

ii) If $(g \circ f)$ is one-to-one then f is one-to-one. **(04 Marks)**

4 c. Using characteristic function, prove that $(A \Delta B) \Delta C = A \Delta (B \Delta C)$. **(06 Marks)**

d. Find the minimal and maximal element of the poset. **(04 Marks)**

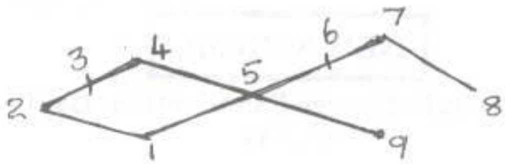

Fig. Q4 (d)

5 a. A function $f : A \to B$ is invertible if and only if it is one-to-one and onto. **(06 Marks)**

b. Let A = {1, 2, 3, 4, 5, 6, 7, 8} compute the products,
i) (2 5 8 6) ∘ (3 8 4) ii) (2 4) ∘ (3 5 7 1) ∘ (1 3 5 7) **(06 Marks)**

c. Let A = {1 2 3 4 5 6}. Determine the values of n such that $P^n = I_A$ for the following permutations $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{bmatrix}$. **(04 Marks)**

d. Let A = B = C = R and $f : A \to B$ and $g : B \to C$ be defined by $f(a) = 3a - 1$ and $g(b) = b^2 + 1$. Find i) $(g \circ f)(-2)$ ii) $(f \circ f)y$. **(04 Marks)**

6 a. Prove that a group G in which every element is its own inverse is abelian. **(05 Marks)**

b. Define a sub group. Let G be a group and $G_1 = \{x \in G \mid xy = yx \text{ for } y \in G\}$. Prove that G_1 is a sub group of G. **(05 Marks)**

c. Let H = {a, c, e} compute the left cosets of the elements of G = {a b c d e f} with respect to H. **(05 Marks)**

d. State and prove Lagrange's theorem. **(05 Marks)**

7 a. The generator matrix for an encoding function $E : Z_2^3 \to Z_2^6$ is given by $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$. Find the code words assigned to 110 to 010. Also obtain the associated parity check. **(06 Marks)**

b. Define a group Homomorphism and group Isomorphism. Let f be a homomorphism from a group G_1 and group G_2. Prove that,
i) If e_1 is the identity in G_1 and e_2 is the identity in G_2 then $f(e_1) = e_2$.
ii) $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G$. **(06 Marks)**

c. Prove that the set Z with binary operation ⊕ and ⊙ defined by $x \oplus y = x + y - 1$, $x \odot y = x + y - xy$ is a commutative ring with unity. **(08 Marks)**

8 a. Define power set and give the power set of the following :
i) {a, {b}} ii) {1, φ, {φ}} **(05 Marks)**

b. Identify the following recurrence relation as linear homogeneous or not. If the relation is a linear homogeneous, give its degree.
i) $a_n + 2a_{n-1} + a_{n-2} = 0$ ii) $d_n = \sqrt{d_{n-1} + d_{n-2}}$
iii) $c_n = c_{n+1}^2 + c_{n-2}^2$ iv) $e_n - 5e_{n-1} + 6e_{n-3} - e_{n-5} = 0$
v) $b_n = b_{n-1} + 5$ **(05 Marks)**

c. Explain the relation 'R' on a set A is i) Reflexive ii) Symmetric iii) Transitive iv) Antisymmetric v) Irreflexive. **(05 Marks)**

d. Define i) maximal element ii) minimal element iii) greatest element iv) least element v) lattice. **(05 Marks)**

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NEW SCHEME

Third Semester B.E. Degree Examination, July 2007
CS / IS
Discrete Mathematical Structures

Time: 3 hrs.] [Max. Marks:100

Note : ***Answer any FIVE full questions.***

1 a. For any three sets A, B, C prove that
$$(A-B)-C = A-(B\cup C) = (A-C)-(B-C)$$ **(07 Marks)**

b. A professor has two dozen introductory textbooks on computer science and is concerned about their coverage of the topics (A) compiler, (b) data structures and (C) operating systems. The following data are the numbers of books that contain material on these topics :
$|A| = 8,\ |B| = 13,\ |C| = 13,\ |A\cap B| = 5,\ |A\cap C| = 3,\ |B\cap C| = 6,\ |A\cap B\cap C| = 2$
i) How many of the textbooks include material on exactly one of these topics?
ii) How many do not deal with any of the topics?
iii) How many have no material on compilers? **(07 Marks)**

c. For all $n \in Z^+$ show that if $n \geq 24$, then n can be written as a sum of $5'^s$ and or $7'^s$. **(06 Marks)**

2 a. Define Tautology. Prove that, for any propositions p, q, r, the compound proposition $\{p\to(q\to r)\}\to\{(p\to q)\to(p\to r)\}$ is a tautology. **(05 Marks)**

b. Prove the following logical equivalences without using truth tables :
i) $p\vee[p\wedge(p\vee q)]\Leftrightarrow p$
ii) $[(\neg p\vee\neg q)\to(p\wedge q\wedge r)]\Leftrightarrow p\wedge q$ **(05 Marks)**

c. For any statements p, q prove that
i) $\neg(p\downarrow q)\Leftrightarrow(\neg p\uparrow\neg q)$
ii) $\neg(p\uparrow q)\Leftrightarrow(\neg p\downarrow\neg q)$ **(05 Marks)**

d. Test the validity of the following argument :
I will become famous or I will not become a musician
I will become a musician. Therefore, I will become famous. **(05 Marks)**

3 a. Write down the following propositions in symbolic form and find its negation.
i) If all triangles are right angled, then no triangle is equiangular.
ii) For all integers n, if n is not divisible by 2, then n is odd. **(07 Marks)**

b. Prove that the following argument is valid :
$$\forall x[p(x)\to q(x)]$$
$$\forall x[q(x)\to r(x)]$$
$$\overline{\therefore\ \forall x[p(x)\to r(x)]}$$
where p(x), q(x) and r(x) are open statements that are defined for a given universe. **(07 Marks)**

c. Provide a proof by contradiction for the following :
For every integer n, if n^2 is odd, then n is odd. **(06 Marks)**

Contd.... 2

4 a. Define Cartesian product of two sets. For A, B, $C \subseteq U$, prove that

$$A \times (B - C) = (A \times B) - (A \times C)$$ **(05 Marks)**

b. Let A = {1, 2, 3, 4} an R be a relation on A defined by aRb if and only if "a divides b". Write down the relation matrix M(R) and draw its digraph. **(05 Marks)**

c. For A = {1, 2, 3, 4}, let R and S be the relations on A defined by R = {(1, 2) (1, 3) (2, 4) (4, 4)} and S = {(1, 1) (1, 2) (1,3) (2, 3) (2, 4)}. Find R_oS, S_oR, R^2, S^2 and R^3 **(05 Marks)**

d. Let A = {a, b, c, d, e}. Consider the partition P = {{a, b}, {c, d}, {e}} of A. Find the equivalence relation inducing this partition. **(05 Marks)**

5 a. If R is an equivalence relation on a set A, and x, y $\in$ A, then prove that

i) $x \in [x]$

ii) x R y if and only if $[x] = [y]$

iii) $[x] = [y]$ or $[x] \cap [y] = \phi$ **(07 Marks)**

b. Define a partially ordered set. The directed graph for a relation R on a set A = {1, 2, 3, 4} is as shown below

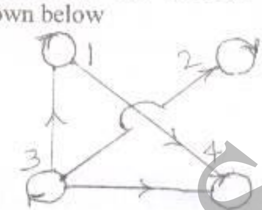

Fig.5(b)

i) Verify that (A, R) is a poset and find its Hasse diagram.

ii) Topologically sort (A, R) **(07 Marks)**

c. Let (A, R) be a poset with $B \subseteq A$. Define

i) Lower bound of B

ii) Upper bound of B

iii) Greatest lower bound of B

iv) Least upper bound of B. **(06 Marks)**

6 a. Define

i) One-to-one function

ii) On to function

Give one example each.

Let f, g, h be functions from Z to Z defined by

$$f(x) = x - 1, \quad g(x) = 3x, \quad h(x) = \begin{cases} 0, & \text{if } x \text{ is even} \\ 1, & \text{if } x \text{ is odd} \end{cases}$$

Verify that $f_o(g_oh) = (f_og)_oh$. **(07 Marks)**

b. Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of the set A = {1, 2, 3, 4, 5, 6}.

i) Write p as a product of disjoint cycles.

ii) Compute p^{-1}

iii) Compute p^2 and p^3

iv) Find the smallest integer R such that $p^R = I_A$ **(07 Marks)**

c. Define characteristic function. For any sets A, B $\subseteq$ U, prove that :

i) $f_{\overline{A}}(x) = 1 - f_A(x)$

ii) $f_{A\cap B}(x) = f_A(x) \cdot f_B(x)$ **(06 Marks)**

7 a. Define Abelian group. Prove that a group G is abelian if and only if for all a, b $\in$ G, $(a, b)^{-1} = a^{-1} b^{-1}$ **(06 Marks)**

b. Define Subgroup. If H, K are subgroups of a group G, prove that H $\cap$ K is also a subgroup of G. Is H $\cup$ K a subgroup of G? Justify your answer. **(06 Marks)**

c. Define left and right cosets. State and prove Lagrange's theorem. **(08 Marks)**

8 a. Define a ring. In a ring (R, +, •), for all a, b $\in$R prove that

i) $a \cdot 0 = 0 \cdot a = 0$

ii) $a(-b) = (-a)b = -(ab)$ **(07 Marks)**

b. Define group Homomorphism. Let f be a homomorphism from a group G_1 to a group G_2. Prove that

i) If e_1 is the identity in G_1 and e_2 is the identity in G_2 then $f(e_1) = e_2$.

ii) $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G_1$ **(07 Marks)**

c. Define a group code. Consider the encoding function $E : Z_2^2 \to Z_2^6$ of the triple repetition code where E(00) = 000000, E(10) = 101010, E(01) = 010101 E(11) = 111111. Prove that $C = E(Z_2^2)$ is a group code. **(06 Marks)**