USN [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ] **06CS34**

Third Semester B.E. Degree Examination, June / July 08

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions, choosing at least TWO from each part.***

PART - A

1 a. Define the following terms and give an example for each i) Set ii) Proper subset iii) Power set iv) Empty set v) Venn diagram. **(05 Marks)**

b. Using the laws of set theory, simplify each of the following

i) $A \cap (B-A)$ ii) $\overline{\overline{(A \cup B) \cap C} \cup \overline{B}}$ **(05 Marks)**

c. In a class of 30 students, 15 take arts, 8 take science, 6 take commerce, 3 take all the three courses. Show that 7 or more students take none of the course. **(05 Marks)**

d. A problem is given to four students A, B, C, D whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ respectively. Find the probability that the problem is solved. **(05 Marks)**

2 a. Let p, q be primitive statements for which the implication $p \rightarrow q$ is false. Determine the truth values for each of the following. i) $p \wedge q$ ii) $\neg p \vee q$ iii) $q \rightarrow p$ iv) $\neg q \rightarrow \neg p$. **(05 Marks)**

b. Verify that $[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \Leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)]$, for primitive statements p, q, and r. **(05 Marks)**

c. Prove the following logical equivalence using the laws of logic : $(\neg p \vee \neg q) \wedge (F_0 \vee P) \wedge P$. **(05 Marks)**

d. Establish the validity of the following argument using the rules of Inference.

$[p \wedge (p \rightarrow q) \wedge (s \vee r) \wedge (r \rightarrow \neg q)] \rightarrow (s \vee t)$. **(05 Marks)**

3 a. What are the bound variables and free variables? Identify the same in each of the following expressions : i) $\forall_y \exists_z [\cos(x+y) = \sin(z-x)]$ ii) $\exists_x \exists_y [x^2 - y^2 = z]$. **(05 Marks)**

b. Let p(x) be the open statement "$x^2 = 2x$", where the universe comprises all integers. Determine whether each of the following statements is true or false. i) P(0) ii) P(1) iii) P(2) iv) P(-2) v) $\exists x\ P(x)$. **(05 Marks)**

c. Provide the steps and reasons to establish the validity of the argument :

$$\begin{array}{l} \forall x[p(x) \rightarrow (q(x) \wedge r(x))] \\ \underline{\forall x[p(x) \wedge s(x)]} \\ \therefore \forall x[r(x) \wedge s(x)] \end{array}$$

(05 Marks)

d. Give a direct proof for each of the following

i) For all integers K and l, if k, l are both even, then k+l is even.

ii) For all integers k and l, if k, l are even, then k.l is even. **(05 Marks)**

4 a. Prove by Mathematical Induction that for every positive integer n, n 1, 2^{n-1}. **(07 Marks)**

b. Apply backtracking technique to obtain an explicit formula for the sequence, defined by the recurrence relation $b_n = 2b_{n-1} + 1$ with initial condition $b_1 = 7$. **(06 Marks)**

c. Solve the linear recurrence relation $a_n = 4a_{n-1} + 5\ a_{n-2}$ with $a_1 = 2$, $a_2 = 6$. **(07 Marks)**

PART – B

5 a. Define the following terms and give an example for each i) Reflexive ii) Irreflexive iii) antisymmetric iv) Transitive v) partition set. **(05 Marks)**

b. For A = {1, 2, 3, 4} and R = { (1, 2), (1, 3), (2, 4), (4,4)}. Compute i) R^2 ii) R^3 iii) R^∞ iv) M_R v) $(M_R)^t$. **(05 Marks)**

c. If $A = A_1 \cup A_2 \cup A_3$, where $A_1 = \{1,2\}$, $A_2 = \{2,3,4\}$ and $A_3 = \{5\}$, define relation R on A by xRy if x and y are in the same subset A_i, for $1 \le i \le 3$. Is R an equivalence relation? **(05 Marks)**

d. Draw the diagraph and Hasse diagram representing the positive divisors of 36. **(05 Marks)**

6 a. For each of the following function, determine whether it is one-to-one and determine its range.
i) $f : Z \to Z$, $f(x) = 2x+1$ ii) $f : Q \to Q$, $f(x) = 2x+1$ iii) $f : Z \to Z$, $f(x) = x^3 - x$
iv) $f : R \to R$, $f(x) = e^x$. v) $f: [0, \pi] \to R$, $f(x) = \sin x$. **(05 Marks)**

b. State the pigeonhole principle. Let ABC be an equilateral triangle with AB = 1. Show that if we select five points in the interior of this triangle, there must be atleast two whose distance apart is less than ½. **(05 Marks)**

c. Define function composition and let A = {1, 2, 3, 4}, B = {a, b, c} and C = {w, x, y, z} with $f : A \to B$ anf $g : B \to C$, given by f = {(1, a), (2, a), (3, b), (4, c) and g = {(a, x), (b, y), (c, z)}. For each of the element of A find gof. **(05 Marks)**

d. Let $f, g : z^+ \to z^+$ where for all $x \in z^+$, $f(x) = x + 1$ and $g(x) = \max\{1, x-1\}$, the maximum of 1 and x-1. i) What is the range of f? ii) Is f an onto function? iii) Is the function f one – to – one. iv) What is the range of g? v) Is g an onto function. **(05 Marks)**

7 a. Define i) Group ii) Subgroup iii) homomorphism iv) Cyclic group v) Coset. **(05 Marks)**

b. Prove that if G is a finite group of order n with H a subgroup of order m, then m divides n. **(05 Marks)**

c. Let $G = S_4$, the symmetric group on four symbols and let H be the subset of G where

$$H = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} \right\}.$$

Construct a table to show that H is an abelian group of G. **(05 Marks)**

d. If G is a group, prove that for all a, b, ∈ G, i) $(a^{-1})^{-1} = a$ ii) $(ab)^{-1} = b^{-1} a^{-1}$. **(05 Marks)**

8 a. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(R)$, prove that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a unit of this ring if and only if $ad - bc \neq 0$. **(05 Marks)**

b. Let (R, +, .) be a commutative ring and let z denote the zero element of R. for a fixed element $a \in R$, define $N(a) = \{r \in R \mid ra = Z\}$. Prove that N(a) is an ideal of R. **(05 Marks)**

c. Construct a decoding table (with syndromes) for the code given by the generator

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$ **(05 Marks)**

d. Prove that for all $n \in N$, $10^n \equiv (-1)^n \pmod{11}$. **(05 Marks)**

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Third Semester B.E. Degree Examination, Dec. 07 / Jan. 08
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note : *Answer any FIVE full questions.*

1 a. Using Venn diagram, prove that, for any sets A, B and C $\overline{\{(A \cup B) \cap C\} \cup \overline{B}} = \overline{B} \cap C$. **(05 Marks)**

b. State and prove De Morgan's Laws of set theory. **(04 Marks)**

c. In a survey of 260 college students, the following data were obtained:
64 had taken a mathematics course, 94 had taken a computer science course, 58 had taken a business course, 28 had taken both a mathematics and a business course, 26 had taken both a mathematics and a computer science course, 22 had taken both a computer science and a business course, and 14 had taken all three types of courses.
i) How many of these students had taken none of the three courses?
ii) How many had taken only a computer science courses? **(06 Marks)**

d. Prove, by mathematical induction $1.3 + 2.4 + 3.5 + ...n(n+2) = \frac{n(n+1)(2n+7)}{6}$. **(05 Marks)**

2 a. If statement q has the truth value 1, determine all truth value assignments for the primitive statements p, r and s for which the truth value of the statement:
$(q \to [(\neg p \vee r) \wedge \neg s]) \wedge [\neg s \to (\neg r \wedge q)]$ is 1. **(04 Marks)**

b. Define tautology. Show that $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is a tautology by constructing truth table. **(05 Marks)**

c. Simplify the compound statement $\neg[\neg((p \vee q) \wedge r) \vee \neg q]$ using laws of logic. Mention the reasons. **(05 Marks)**

d. Write the following argument in symbolic form and then establish its validity:
If Rochelle gets the supervisor's position and works hard, then she'll get a raise. If she gets the raise, then she'll buy a new car. She has not purchased a new car. Therefore, either Rochelle did not get the supervisor's position or she did not work hard. **(06 Marks)**

3 a. Define an open statement. Write the negation of the statement: If k, m, n are any integers where k – m and m – n are odd then k – n is even. **(07 Marks)**

b. For the universe of all integers, define the following open statements:
p(x): x>0, q(x): x is even, r(x): x is a perfect square, s(x): x is divisible by 4 and t(x): x is divisible by 5.
Write the following statements in symbolic form and determine whether each of the statements is true or false. For each false statement, provide a counter example.
i) Atleast one integer is even ii) There exists a positive integer that is even iii) If x is even, then x is not divisible by 5 iv) If x is even and x is a perfect square, then x is divisible by 4. **(07 Marks)**

c. Give: i) A direct proof, ii) An indirect proof and iii) Proof by contradiction, for the following statement, "If n is an odd integer, then n+9 is an integer". **(06 Marks)**

4 a. Let A = {1, 2, 3, 4, 6} and 'R' be a relation on 'A' defined by aRb if and only if 'a' is multiple of 'b': i) Write down R as a set of ordered pairs ii) Represent R as a matrix iii) Draw the digraph of R. **(06 Marks)**

b. Let A = {1, 2, 3, 4, 5} × {1, 2, 3, 4, 5}, and define R on A by
$(x_1, y_1)R(x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$.
i) Verify that R is an equivalence relation on A.
ii) Determine the equivalence classes [(1, 3)], [(2, 4)] and [(1, 1)]. **(08 Marks)**

c. Define a poset. Consider the Hasse diagram of a poset (A, R) given below in fig.Q4(c). If B = {c, d, e}, find (if they exist).
i) The least upper bound of B ii) The greatest lower bound of B. **(06 Marks)**

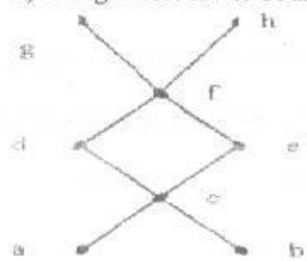

Fig.Q4(c)

5 a. Let S = {-1, 0, 2, 4, 7}. Find f(S) if i) $f(x) = 1$, ii) $f(x) = 2x+1$, iii) $f(x) = \left\lceil \frac{x}{5} \right\rceil$,
iv) $f(x) = \left\lfloor \frac{(x^2+1)}{3} \right\rfloor$. **(06 Marks)**

b. Let A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6}
i) How many functions are there from A to B? How many of these are one-to-one? How many are onto?
ii) How many functions are there from B to A? How many of these are onto? How many are one-to-one? **(06 Marks)**

c. Let A = B = R. Determine $\pi_A(D)$ and $\pi_B(D)$ for each of the following sets $D \subseteq A \times B$.
i) $D = \{(x, y)/x = y^2, 0 \le y \le 2\}$ ii) $D = \{(x, y)/y = \text{Sin}x, 0 \le x \le \pi\}$. **(08 Marks)**

6 a. Let $f : R \to R$ be defined by, $f(x) = \begin{cases} 3x-5, & x > 0 \\ -3x+1, & x \le 0 \end{cases}$. Find $f^{-1}(0)$, $f^{-1}(1)$, $f^{-1}(3)$ and $f^{-1}([-5, 5])$. **(06 Marks)**

b. Prove that a function $f : A \to B$ is invertible if and only if it is one-to-one and onto. **(08 Marks)**

c. Prove that if 151 integers are selected from {1, 2, 3, ..., 300}, then the selection must include two integers x, y where x | y or y | x. **(06 Marks)**

7 a. Define the binary operation ∘ on Z by $x \circ y = x + y + 1$. Verify that (Z, ∘) is an Abelian group. **(05 Marks)**

b. Let (G, •) and (H, *) be two groups with respective identities e_G, e_H. If $f : G \to H$ is a homomorphism, then prove that i) $f(e_G) = e_H$ ii) $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G$
iii) f(S) is a subgroup of H for each subgroup S of G. **(08 Marks)**

c. Define cyclic group. Prove that every subgroup of a cyclic group is cyclic. **(07 Marks)**

8 a. Define group code. Let $E : Z_2^m \to Z_2^n$, m < n be the encoding function given by a generator matrix G or the associated parity – check matrix H. Prove that $C = E(Z_2^m)$ is a group code. **(10 Marks)**

b. Define a ring and an integral domain. Let R be a commutative ring with unity. Prove that R is an integral domain if and only if for all a, b, c ∈ R, where a ≠ z, (additive identity) $ab = ac \Rightarrow b = c$. **(10 Marks)**

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