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**Third Semester B.E. Degree Examination, Dec.08/Jan.09**

**Discrete Mathematical Structure**

Time: 3 hrs.

Max. Marks:100

**Note: Answer any FIVE full questions  
choosing at least two from each part.**

**Part A**

- 1 a. Define Tautology:  
Show that  $[(p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))] \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$  is tautology using laws. (08 Marks)
- b. Define dual of logical statement. Write dual of logical statement,  
 $(P \vee T_0) \wedge (q \vee F_0) \vee (r \wedge s \wedge T_0)$ . (04 Marks)
- c. Write truth values for NAND and NOR. (02 Marks)
- d. Test validity of the following statement:  
i) If there is strike by student, the examination will be postponed,  
The Exam was not postponed  
 $\therefore$  There were no strike by student  
ii) If Ravi studies, then he will pass DMS. If Ravi does not play cricket, then he will study  
Ravi failed in DMS  
 $\therefore$  Ravi played cricket (06 Marks)
- 2 a. Define converse, inverse, contrapositive of implication, hence find converse, inverse, contrapositive for  $\forall x, (x > 3) \rightarrow (x^2 > 9)$  where universal set is R. (04 Marks)
- b. For any two odd integer mandn show that,  
i)  $m + n$  is even. ii)  $mn$  is odd. (06 Marks)
- c. Using quantifier method find whether following argument is valid,  
If a triangle has two equal sides, then it is isoceses. If a triangle is isoceses, then it has two equal angles.  
The triangle ABC does not have two equal angles  
 $\therefore$  ABC does not have two equal sides (10 Marks)
- 3 a. Find A and B if  $A \cup B = \{1, 2, 4, 5, 7, 8, 9, 10\}$ ,  $A \cap B = \{2, 4, 7\}$ ,  $A - B = \{1, 8\}$ . (02 Marks)
- b. Using laws show that,  $(A \cap B) \cup (A \cap B \cap \bar{C} \cap D) \cup (\bar{A} \cap B) = B$ . (06 Marks)
- c. 21 students took Maths exam having 3 questions and all of them answered at least one question of 5 fail to answer 1<sup>st</sup> question, 6 fail to answer 2<sup>nd</sup> question 7 fail to answer 3<sup>rd</sup> question. If 9 answered all 3 question, find how many answered exactly one question. (08 Marks)
- d. Find probability of two persons A and B contradicting when they narrate same story, given A speaks 60% true and B speaks 20% false. (04 Marks)
- 4 a. A sequence  $\{a_n\}$  defined by  $a_1 = 4$ ,  $a_n = n + a_{n-1}$  for  $n \geq 2$ .  
Show that explicit expression of  $a_n = 3 + \frac{1}{2}(n^2 + n)$ . (07 Marks)
- b. Prove by mathematical induction,  $O[P(A)] = 2^n$ . If  $O(A) = n$  where A is given set. (07 Marks)
- c. Let  $R = \{(1, 2), (1, 3), (2, 4), (3, 2)\}$  be relation on  $A = \{1, 2, 3, 4\}$  find  $M(R)$  and  $[M(R)]^2$  hence find  $R^2$ . (06 Marks)

**Part B**

- a. Define equivalence relation and equivalence class with one example. (07 Marks)  
b. For  $A = \{a, b, c, d, x, y, z\}$ , define equivalence relation hence find equivalence class. Also find portion of A. (07 Marks)

c. Find  $A \times B$ ,  $A \times (B \cup C)$ ,  $(A \cap B) \times C$  of  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  $C = \{2, 4, 6\}$ . (06 Marks)

a. Define a function. Prove that function  $f : A \rightarrow B$  is invertible. If it is one-one and onto. (07 Marks)

b. Define sterling number of 2<sup>nd</sup> kind. If  $|A| = 7$ ,  $|B| = 4$  find number of onto function from A to B. Hence find  $S(7, 4)$ . (07 Marks)

c. If  $f(x) = x-1$ ,  $g(x) = 3x$   $h(x) = \begin{cases} 0 & x \text{ even} \\ 1 & x \text{ odd} \end{cases}$ . Show that  $f \circ (g \circ h) = (f \circ g) \circ h$ . (06 Marks)

a. Let G be set non-zero real.  $a * b = \frac{ab}{2}$  for  $a, b \in G$ . Show that  $(G, *)$  is Abelian group. (06 Marks)

b. Define sub group. If H, K are subgroup of G. Prove that  $H \cap K$  is also subgroup. Is  $H \cup K$  is subgroup of G. Justify the answer. (06 Marks)

c. Define left and right cosets. State and prove Lagrange's theorem. (08 Marks)

a. The Generator matrix for an encoding function,  $E : Z_2^3 \rightarrow Z_2^6$  is given by,

$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ . Find the code word assigned to 110 to 010. Also find associated parity check. (08 Marks)

b. Define: i) Hamming metric ii) The sphere of radius k centered at X. Give example in each case. (06 Marks)

c. Define ring with an example. (06 Marks)