USN

06CS34

Third Semester B.E. Degree Examination, May/June 2010
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Define power set of a set. Find the power sets of the following set : $A = \{0, \phi, \{\phi\}\}$. **(04 Marks)**

b. Using laws of set theory, prove that $(A \cap B) \cup [B \cap ((C \cap D) \cup (C \cap \overline{D}))] = B \cap (A \cup C)$. **(06 Marks)**

c. An integer is selected at random from 3 through 7 inclusive. If A is the event that a number divisible by 3 is chosen and B is the event that the number exceeds 10, determine $P_r(A)$, $P_r(B)$, $P_r(A \cap B)$ and $P_r(A \cup B)$ **(04 Marks)**

d. A professor has two dozen textbooks on computer science and is concerned about their coverage of topics : (A) compilers, (B) data structures, and (C) operating systems. Following are the numbers of books that contain material on these topics : $|A| = 8$, $|B| = 13$, $|C| = 13$, $|A \cap B| = 5$, $|A \cap C| = 3$, $|B \cap C| = 6$, $|A \cap B \cap C| = 2$.

i) How many of the textbooks include material on exactly one of these topics?

ii) How many do not deal with any of the topics? **(06 Marks)**

2 a. Define the following : i) Proposition ii) Tautology iii) Contradiction.

Determine whether the following compound statement is tautology or not :

$$[(p \to q) \wedge (q \to r)] \to (p \to r)$$ **(08 Marks)**

b. Using rules of inference, show that the following argument is valid : **(06 Marks)**

$$\begin{array}{l} p \\ p \to q \\ s \vee r \\ \underline{r \to \neg q} \\ \therefore s \vee t \end{array}$$

c. Simplify the following switching network, (without using the truth table). **(06 Marks)**

3 a. Establish the validity of the following argument: **(10 Marks)**

$$\begin{array}{l} \forall_x [p(x) \vee q(x)] \\ \exists_x \neg p(x) \\ \forall_x [\neg q(x) \vee r(x)] \\ \underline{\forall_x [s(x) \to \neg r(x)]} \\ \therefore \exists_x \neg s(x) \end{array}$$

b. For the following statements the universe comprises all nonzero integers. Determine the truth value of each statement :

i) $\exists_x \exists_y [xy = 1]$ ii) $\exists_x \forall_y [xy = 1]$ iii) $\forall_x \exists_y [xy = 1]$ **(06 Marks)**

c. Negate and simplify each of the following : i) $\forall_x [p(x) \wedge \neg q(x)]$

ii) $\exists_x [p(x) \vee q(x)] \to r(x)$ **(04 Marks)**

4 a. Define the following : i) Well-ordering principle

ii) Principle of mathematical induction. **(04 Marks)**

b. By the principle of mathematical induction, prove that :

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$ **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

4 c. Give a recursive definition for each of the following integer sequence :
i) $c_n = 7n$ ii) $c_n = 2 - (-1)^n$. For $n \in z^+$. **(04 Marks)**

d. For $n \geq 0$ let F_n denote the n^{th} Fibonacci number. Prove that for any positive integer n, $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$. **(06 Marks)**

PART – B

5 a. Define Cartesian product of two sets. For any three non-empty sets A, B, C. Prove that $A \times (B - C) = (A \times B) - (A \times C)$. **(05 Marks)**

b. Define the following : i) Function ; ii) Onto function ; iii) One – to – one. Let $f: z \to z$ be defined by f(a) = a + 1 for all a ∈ z. Find whether f is one-to-one correspondence or not. **(05 Marks)**

c. State the pigeonhole principle. Show that if any seven numbers from1 to 12 are chosen, then two of them will add to 13. **(05 Marks)**

d. Let $f, g: R \to R$ be defined by f(x) = 2x + 5, $g(x) = \left(\frac{1}{2}\right)(x - 5)$. Show that f and g are invertible. **(05 Marks)**

6 a. Let A = {1, 2, 3, 4}, Let R = {(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)} be the relation on A. Determine whether the relation R is reflexive, irreflexive, symmetric, antisymmetric or transitive. **(05 Marks)**

b. Let A = {1, 2), B = {m, n, p} and C = {3, 4}. Let R_1 = {(1, m), {(1, n), {(1, p)}, R_2 = {(m, 3), (m, 4), (p, 4)} and R_3 = {(m, 3), (m, 4), (p, 3)}. Prove that :
$$R_1 \circ (R_2 \cap R_3) \subseteq (R_1 \circ R_2) \cap (R_1 \circ R_3).$$ **(05 Marks)**

c. Let A = {a, b, c}, B = P(A), where P(A) is the power set of A. Let R is a subset relation on A. Draw the Hasse diagram of the poset (B, R). **(05 Marks)**

d. Let A = {2, 3, 4, 6, 8, 12, 24} and let ≤ denotes the partial order of divisibility, that is x ≤ means x divides y. Let B = {4, 6, 12}. Determine :
i) All upper bounds of B ii) All lower bounds of B
iii) Least upper bound of B iv) Greatest lower bound of B. **(05 Marks)**

7 a. For any group G prove that G is abelian if and only if $(ab)^2 = a^2b^2$ for all a, b, ∈ G. **(05 Marks)**

b. State and prove Lagrange's theorem. **(05 Marks)**

c. A binary symmetric channel has probability p = 0.05 of incorrect transmission. If the word c = 011011101 is transmitted. What is the probability that :
i) Single error occurs, ii) Three errors occur, no two of them consecutive? **(05 Marks)**

d. Determine the minimum distance between the code words, $E: Z_2^3 \to Z_2^6$

$000 \to 000111$ $001 \to 001001$
$010 \to 010010$ $011 \to 011100$
$100 \to 100100$ $101 \to 101010$
$110 \to 110001$ $111 \to 111000$

How many errors can be detected and corrected by this code? **(05 Marks)**

8 a. Construct a decoding table (with syndromes) for the group code given by the generator matrix: $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$. Using this decoding table, decode the following received words :
11110, 11011, 10000, 10101 **(10 Marks)**

b. Determine whether (z, ⊕,⊙) is a ring with the binary operations x ⊕ y = x + y – 7, x ⊙ y = x + y – 3xy for all x, y ∈ z. **(10 Marks)**

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