USN

06CS42

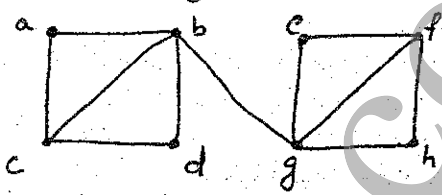
Fourth Semester B.E. Degree Examination, May/June 2010
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Let G = (V, E) be the undirected graph in the Fig.1(a). How many paths are there in G from a to h? How many of these paths have a length 5? **(07 Marks)**

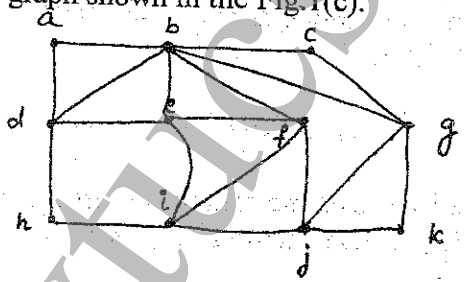
Fig.1(a).



b. Let G = (V, E) be an undirected graph, where $|V| \geq 2$. If every induced subgraph of G is connected, can we identify the graph G? **(06 Marks)**

c. Find an Euler circuit for the graph shown in the Fig.1(c). **(07 Marks)**

Fig.1(c).



2 a. Show that when any edge is removed from K_5, the resulting subgraph is planar. Is this true for the graph $K_{3,3}$? **(07 Marks)**

b. Nineteen students in a nursery school, play a game each day, where, they hold hands to form a circle. For how many days can they do this, with no student holding hands with the same playmate twice? **(07 Marks)**

c. Define chromatic number. What is chromatic polynomial? State the decomposition theorem for chromatic polynomials. **(06 Marks)**

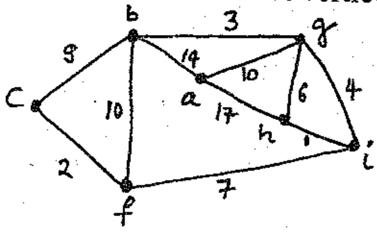
3 a. A classroom contains 25 microcomputers, that must be connected to a wall socket that has four outlets. Connections are made by using extension cords, that have four outlets each. What is the least number cords needed to get these computers set up for class use? **(07 Marks)**

b. Explain the steps in the merge sort algorithm. **(06 Marks)**

c. Using the weights 2, 3, 5, 10, 10, show that the height of the Huffman tree for a given set of weights is not unique. **(07 Marks)**

4 a. Apply Dijkstra algorithm to the weighted graph G = (V, E) shown in Fig.4(a) and determine the shortest distance from vertex a to each of the other vertices in the graph. **(07 Marks)**

Fig.4(a)



Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. Use Prim's algorithm to generate an optimal tree for the graph, shown in Fig.4(b). **(06 Marks)**

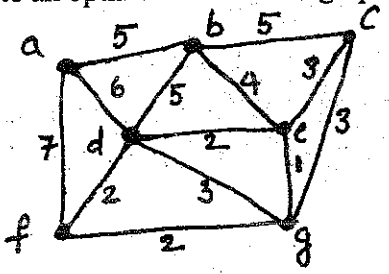


Fig.4(b).

c. Let f be a flow in a network N = (V, E). If C = (P, $\overline{P}$) is any cut in N, then prove that val (f) cannot exceed C (P, $\overline{P}$). **(07 Marks)**

5 a. In a certain implementation of the programming language Pascal, an identifier consists of a single letter or a letter followed by upto seven symbols, which may be letters or digits. (26 letters, 10 digits). There are 36 reserved words. How many distinct identifiers are possible in this version of Pascal? **(07 Marks)**

b. How many bytes contain i) Exactly two 1's ; ii) Exactly four 1's ; iii) Exactly six 1's and iv) At least six 1's? **(07 Marks)**

c. In how many ways can 10 (identical) dimes be distributed among five children i) If there are no restrictions ; ii) Each child gets at least one dime ; iii) The oldest child gets at least 2 dimes. **(06 Marks)**

6 a. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3, 5. **(07 Marks)**

b. In how many ways can one arrange the letters in CORRESPONDENTS so that i) There is no pair of consecutive identical letters ; ii) There are exactly two pairs of consecutive identical letters. **(07 Marks)**

c. For the positive integers 1, 2, 3, 4, there are n derangements. Define derangements. What is the value of n? **(06 Marks)**

7 a. Give the generating function for :

i) 1, 1, 1, 1,……..1, 1, 1…. all terms are 1

ii) 1, 1, 1, 1,……..1, 0, 0, 0….. first terms are 1, others are 0

iii) 0, 1, 2, 3……. **(06 Marks)**

b. Find the generating function for $P_d(n)$, the number of partitions of a positive integer n into distinct summands. What is $P_d(6)$ = ? **(07 Marks)**

c. In each of the following, the function f(x) is the exponential generating function for the sequence a_0, a_1, a_2……., whereas the function g(x) is the exponential generating function for the sequence b_0, b_1, b_2……. Express g(x) in terms of f(x) if

i) $b_3 = 3$ and $b_n = a_n$, $n \in N$, $n \neq 3$.

ii) $b_1 = 2$, $b_2 = 4$, and $b_n = 2a_n$, $n \in N$, $n \neq 1, 2$. **(07 Marks)**

8 a. Solve the following recurrence relation :

$a_n = 5a_{n-1} + 6a_{n-2}$ $n \geq 2$ $a_0 = 1, a_1 = 3$. **(10 Marks)**

b. Solve the following recurrence relation:

$a_{n+1} - 2a_n = 2^n$, $n \geq 0$ $a_0 = 1$. **(10 Marks)**

c. Solve the following recurrence relation using the method of generating functions :

$a_{n+2} - 5a_{n+1} + 6a_n = 2$, $n \geq 0, a_0 = 3, a_1 = 7$.

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USN

06CS42

Fourth Semester B.E. Degree Examination, December 2010
Graph Theory and Combinatories

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least two questions from each part.***

PART – A

1 a. Define: i) Connected graph ii) Spanning subgraph, and iii) Complement of a graph. Give one example for each. **(06 Marks)**

b. Explain, with an example, graph isomorphism. Show that in a graph G, the number of odd degree vertices is even. **(07 Marks)**

c. Write a note on "Konigsberg-bridge problem". **(07 Marks)**

2 a. Define complete bipartite graph. Prove that Kuratowski's second graph $K_{3,3}$, is nonplanar. **(06 Marks)**

b. Show that in any connected planar graph with 'n' vertices, 'e' edges and 'f' faces, e-n+2=f (Euler's formula). **(07 Marks)**

c. Define chromatic number and chromatic polynomial. Find the chromatic polynomial for the graph given below: **(07 Marks)**

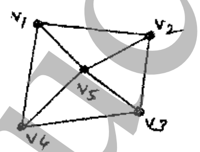

Fig. Q2 (c)

3 a. Define : i) Tree ii) Binary rooted tree, and iii) Prefix code.
Give one example for each. **(06 Marks)**

b. Prove that a tree with 'n' vertices has (n-1) edges. **(07 Marks)**

c. Find all the spanning trees of the graph given below: **(07 Marks)**

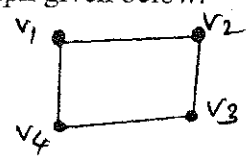

Fig. Q3 (c)

4 a. Define : i) Matching ii) Complete matching and iii) Edge-connectivity (with example). **(06 Marks)**

b. Find a minimal spanning tree using prims algorithm for the weighted graph given below: **(07 Marks)**

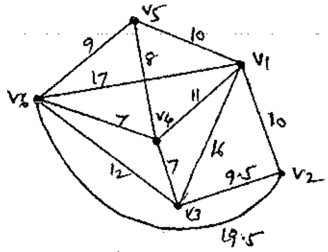

Fig. Q4 (b)

4 c. Find the maximum flow possible between the vertices A and D for the following graph: **(07 Marks)**

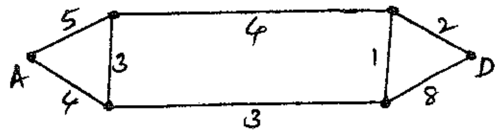

Fig. Q4 (c)

PART – B

5 a. In how many ways can one distribute 10 identical white marbles among six distinct containers? **(06 Marks)**

b. i) How many 9 letter words can be formed using the letters of the word "Difficult"?

ii) A certain question paper contains two parts A and B each having 4 questions. How many different ways a student can answer 5 questions by selecting at least two questions from each part? **(07 Marks)**

c. Let a triangle ABC be equilateral, with AB = 1. Show that if we select 10 points in the interior of this triangle, there must be at least two points, whose distance apart is less than $\frac{1}{3}$. **(07 Marks)**

6 a. In how many ways can the integers 1, 2, 3, ...10 be arranged in a line, so that, no even integer is in its natural place? **(06 Marks)**

b. In how many ways can one arrange the letters in CORRESPONDENTS so that:

i) There are exactly two pairs of consecutive identical letters.

ii) There are at least three pairs of consecutive identical letters. **(07 Marks)**

c. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3 and B_4. The boys B_1 and B_2 do not wish to have the apple, the boy B_3 does not want the banana or mango and B_4 returns the orange. In how many ways the distribution can be made so that no boy is displeased? **(07 Marks)**

7 a. Let $f(x) = (1 + x + x^2)(1 + x)^n$, where $n \in z^+$. Find the coefficient of the following in the expansion of f(x):

i) x^7 ii) x^8 iii) x^r, $0 \le r \le (n+2)$, $r \in z$. **(06 Marks)**

b. In how many ways can 12 oranges be distributed among three children A, B, C so that A gets at least four, B and C get at least two but C gets no more than five? **(07 Marks)**

c. Find the exponential generating function for the number of ways to arrange 'n' letters, $n \ge 0$, selected from each of the following words:

i) HAWAII ii) MISSISSIPPI iii) ISOMORPHISM **(07 Marks)**

8 a. The number of bacteria in a culture is 1000 (approximately) and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. **(06 Marks)**

b. Solve the recurrence relation,

$a_n - 6a_{n-1} + 9a_{n-2} = 0$, $n \ge 2$, given $a_0 = 5$, $a_1 = 12$ **(07 Marks)**

c. Find the generating function for the recurrence relation,

$a_{n+2} - 5a_{n+1} + 6a_n = 2$, $n \ge 0$, with $a_0 = 3$, $a_1 = 7$. Hence solve it. **(07 Marks)**

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