NEW SCHEME

Reg. No.

Third Semester B.E. Degree Examination, January/February 2006

Computer Science/Information Science and Engineering

Discrete Mathematical Structures

Time: 3 hrs.) (Max.Marks : 100

Note: *1. Answer any FIVE full questions.*
2. All questions carry equal marks.

1. (a) Determine the sets A and B, given that $A - B = \{1,2,4\}$, $B - A = \{7,8\}$, and $A \cup B = \{1,2,4,5,7,8,9\}$ **(4 Marks)**

(b) Prove that : $\overline{A \triangle B} = \overline{A} \triangle B = A \triangle \overline{B}$ **(6 Marks)**

(c) The freshman class of a private engineering college has 300 students. It is known that 180 can program in PASCAL, 120 in FORTRAN, 30 in C^{++}, 12 in PASCAL and C^{++}, 18 in FORTRAN and C^{++}, 12 in PASCAL and FORTRAN, and 6 in all three languages. If two students are selected at random, what is the probability that they can i) both program in PASCAL ? ii) both program only in Pascal. **(6 Marks)**

(d) Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(4 Marks)**

2. (a) If statement q has the truth value 1, determine all truth value assignments for the primitive statements p, r and s for which the truth vaue of the statement.

$$[q \rightarrow \{\neg p \vee r) \wedge \neg s\}] \wedge \{\neg s \rightarrow (\neg r \wedge q)\}$$ **(4 Marks)**

(b) Define tautology. Prove that, for any propositions p, q, r, the compound proposition $[(p \rightarrow q) \wedge (q \rightarrow r)]$ is a tautology. **(6 Marks)**

(c) Prove the following logical equivalences without using truth tables :

i) $p \vee [p \wedge (p \vee q)] \Longleftrightarrow p$

ii) $\neg [\neg \{p \vee q) \wedge r\} \vee \neg q] \Longleftrightarrow q \wedge r$ **(6 Marks)**

(d) Test whether the following argument is valid :

If interest rates fall, then the stock market will rise. The stock market will not rise. Therefore the interest rates will not fall. **(4 Marks)**

3. (a) Consider the following open statements with the set of all real numbers as the universe

$p(x) : x \geq 0, \;\; q(x) : x^2 \geq 0$

$r(x) : x^2 - 3x - 4 = 0, \;\; s(x) : x^2 - 3 > 0,$

then find the truth values of the following statement.

i) $\exists\, x\, [p(x) \wedge r(x)]$

ii) $\forall\, x\, [p(x) \rightarrow q(x)]$

iii) $\forall\, x\, [q(x) \rightarrow s(x)]$ **(6 Marks)**

Contd.... 2

(b) Write down the following propositions in symbolic form, and find its negation.

i) For all integers n, if n is not divisible by 2, then n is odd.

ii) All integers are rational numbers and some rational numbers are not integers **(7 Marks)**

(c) For the universe of all people, find whether the following is a valid argument :

All mathematics professors have studied calculus
Ramanujan is a mathematics professor
Therefore Ramanujan has studied calculus. **(7 Marks)**

4. (a) Define cartesian product of two sets. For any three non-empty sets A, B, C prove that $A \times (B - C) = (A \times B) - (A \times C)$ **(6 Marks)**

(b) Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by aRb if and only if a is c multiple of b. Write down the relation matrix $M(R)$ and draw its digraph. **(6 Marks)**

(c) If $A = \{1, 2, 3, 4\}$ and R, S are relations on A defined by $R = \{(1,2), (1,3), (2,4), (4,4)\}$, $S = \{(1,1)\ (1,2)\ (1,3)(1,4), (2,3)\ (2,4)\}$ find RoS, SoR, R^2 and S^2. **(4 Marks)**

(d) Let $A = \{a, b, c, d, e\}$. Consider the partition $P = \{\{a, b\}, \{c, d\}\{e\}\}$ of A. Find the equivalance relation inducing this partition. **(4 Marks)**

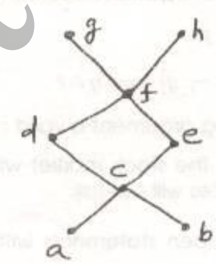
5. (a) Let $A = \{1, 2, 3, 4, 5\}$. Define a relation R on $A \times A$ by $(x_1, y_1)\ R(x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.

i) Verify that R is an equivalence relation on $A \times A$

ii) Determine the equivalence classes $[(1,3)]$, $[(2,4)\ and\ [1,1]]$ **(6 Marks)**

(b) Define a partially ordered set. If R is a relation on the set $A = \{1, 2, 3, 4\}$ defined by xRy if x divides y, prove that (A, R) is a poset. Draw its Hasse diagram. **(7 Marks)**

(c) Let A be a set and $B \subseteq A$. Define i) Least upper bound of B ii) Greatest lower bound of B. Consider the poset whose Hasse diagram is shown below Find LUB and GLB of the set $B = \{c, d, e\}$



(7 Marks)

6. (a) Prove that, a function $f : A \longrightarrow B$ is invertible if and only if it is one-one and onto. **(6 Marks)**

(b) Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of the set $A = \{1, 2, 3, 4, 5, 6\}$.

i) Write p as a product of disjoint cycles

ii) Compute p^{-1}

iii) Compute p^2 and p^3

iv) Find the smallest integer k such that $p^k = I_A$ **(7 Marks)**

Define characteristic funtion. For any sets A, B contained in a universal set U, prove that:

i) $f_{\overline{A}}(x) = 1 - f_A(x)$

ii) $f_{A \cap B}(x) = f_A(x) \cdot f_B(x)$ **(7 Marks)**

Define an abelian group. Prove that a group G is abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$. **(6 Marks)**

Prove that, if H is a non empty subset of a group G, then H is a subgroup of G if and only if

i) for all $a, b \in H,\ ab \in H$ and

ii) for all $a \in H,\ a^{-1} \in H$ **(6 Marks)**

Define left and right cosets. State and prove Lagrange's theorem. **(8 Marks)**

Define a ring. If R is a ring with unity and $a,\ b$ are units in R, prove that ab is a unit in R and that $(ab)^{-1} = b^{-1}a^{-1}$. **(6 Marks)**

If $f : G \longrightarrow H,\ \ g : H \longrightarrow K$ are homomorphisms, prove that the composite function $gof : G \longrightarrow K$, where $(gof)\ (x) = g(f(x))$, is a homomorphism. **(6 Marks)**

Define a group code. Let $E : {Z_2}^m \longrightarrow {Z_2}^n$ be an encoding function given by a generator matrix G or the associated parity - check matrix H. Then prove that $C = E({Z_2}^m)$ is a group code. **(8 Marks)**

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USN

NEW SCHEME

Third Semester B.E. Degree Examination, July 2007
CS / IS
Discrete Mathematical Structures

Time: 3 hrs.] [Max. Marks:100

Note : ***Answer any FIVE full questions.***

1 a. For any three sets A, B, C prove that
$(A - B) - C = A - (B \cup C) = (A - C) - (B - C)$ **(07 Marks)**

b. A professor has two dozen introductory textbooks on computer science and is concerned about their coverage of the topics (A) compiler, (b) data structures and (C) operating systems. The following data are the numbers of books that contain material on these topics :
$|A| = 8,\ |B| = 13,\ |C| = 13,\ |A \cap B| = 5,\ |A \cap C| = 3,\ |B \cap C| = 6,\ |A \cap B \cap C| = 2$
i) How many of the textbooks include material on exactly one of these topics?
ii) How many do not deal with any of the topics?
iii) How many have no material on compilers? **(07 Marks)**

c. For all $n \in Z^+$ show that if $n \geq 24$, then n can be written as a sum of 5's and or 7's. **(06 Marks)**

2 a. Define Tautology. Prove that, for any propositions p, q, r, the compound proposition $\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is a tautology. **(05 Marks)**

b. Prove the following logical equivalences without using truth tables :
i) $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$
ii) $[(\neg p \vee \neg q) \to (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$ **(05 Marks)**

c. For any statements p, q prove that
i) $\neg(p \downarrow q) \Leftrightarrow (\neg p \uparrow \neg q)$
ii) $\neg(p \uparrow q) \Leftrightarrow (\neg p \downarrow \neg q)$ **(05 Marks)**

d. Test the validity of the following argument :
I will become famous or I will not become a musician
I will become a musician. Therefore, I will become famous. **(05 Marks)**

3 a. Write down the following propositions in symbolic form and find its negation.
i) If all triangles are right angled, then no triangle is equiangular.
ii) For all integers n, if n is not divisible by 2, then n is odd. **(07 Marks)**

b. Prove that the following argument is valid :

$$\begin{array}{l} \forall x[p(x) \to q(x)] \\ \forall x[q(x) \to r(x)] \\ \hline \therefore\ \forall x[p(x) \to r(x)] \end{array}$$

where p(x), q(x) and r(x) are open statements that are defined for a given universe. **(07 Marks)**

c. Provide a proof by contradiction for the following :
For every integer n, if n^2 is odd, then n is odd. **(06 Marks)**

Contd.... 2

4 a. Define Cartesian product of two sets. For A, B, C $\subseteq$ U, prove that

$$A \times (B - C) = (A \times B) - (A \times C)$$

(05 Marks)

b. Let A = {1, 2, 3, 4} an R be a relation on A defined by aRb if and only if "a divides b". Write down the relation matrix M(R) and draw its digraph. (05 Marks)

c. For A = {1, 2, 3, 4}, let R and S be the relations on A defined by R = {(1, 2) (1, 3) (2, 4) (4, 4)} and S = {(1, 1) (1, 2) (1,3) (2, 3) (2, 4)}. Find R_oS, S_oR, R^2, S^2 and R^3 (05 Marks)

d. Let A = {a, b, c, d, e}. Consider the partition P = {{a, b}, {c, d}, {e}} of A. Find the equivalence relation inducing this partition. (05 Marks)

5 a. If R is an equivalence relation on a set A, and x, y $\in$ A, then prove that

i) $x \in [x]$

ii) x R y if and only if $[x] = [y]$

iii) $[x] = [y]$ or $[x] \cap [y] = \phi$ (07 Marks)

b. Define a partially ordered set. The directed graph for a relation R on a set A = {1, 2, 3, 4} is as shown below

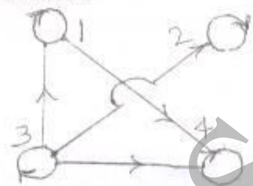

Fig.5(b)

i) Verify that (A, R) is a poset and find its Hasse diagram.

ii) Topologically sort (A, R) (07 Marks)

c. Let (A, R) be a poset with B $\subseteq$ A. Define

i) Lower bound of B

ii) Upper bound of B

iii) Greatest lower bound of B

iv) Least upper bound of B. (06 Marks)

6 a. Define

i) One-to-one function

ii) On to function

Give one example each.

Let f, g, h be functions from Z to Z defined by

$$f(x) = x - 1, \quad g(x) = 3x, \quad h(x) = \begin{cases} 0, & \text{if } x \text{ is even} \\ 1, & \text{if } x \text{ is odd} \end{cases}$$

Verify that $f_o(g_oh) = (f_og)_oh$. (07 Marks)

b. Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of the set A = {1, 2, 3, 4, 5, 6}.

i) Write p as a product of disjoint cycles.

ii) Compute p^{-1}

iii) Compute p^2 and p^3

iv) Find the smallest integer R such that $p^R = I_A$ (07 Marks)

c. Define characteristic function. For any sets A, B $\subseteq$ U, prove that :

i) $f_{\overline{A}}(x) = 1 - f_A(x)$

ii) $f_{A \cap B}(x) = f_A(x) . f_B(x)$ **(06 Marks)**

7 a. Define Abelian group. Prove that a group G is abelian if and only if for all a, b $\in$ G, $(a, b)^{-1} = a^{-1} b^{-1}$ **(06 Marks)**

b. Define Subgroup. If H, K are subgroups of a group G, prove that H $\cap$ K is also a subgroup of G. Is H $\cup$ K a subgroup of G? Justify your answer. **(06 Marks)**

c. Define left and right cosets. State and prove Lagrange's theorem. **(08 Marks)**

8 a. Define a ring. In a ring (R, +, $\bullet$), for all a, b $\in$R prove that

i) $a . 0 = 0 . a = 0$

ii) $a(-b) = (-a)b = -(ab)$ **(07 Marks)**

b. Define group Homomorphism. Let f be a homomorphism from a group G_1 to a group G_2. Prove that

i) If e_1 is the identity in G_1 and e_2 is the identity in G_2 then $f(e_1) = e_2$.

ii) $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G_1$ **(07 Marks)**

c. Define a group code. Consider the encoding function $E : Z_2^2 \rightarrow Z_2^6$ of the triple repetition code where E(00) = 000000, E(10) = 101010, E(01) = 010101 E(11) = 111111. Prove that $C = E(Z_2^2)$ is a group code. **(06 Marks)**

USN | | | | | | | | | | | 06MAT31

Third Semester B.E. Degree Examination, June / July 08
Engineering Mathematics - III

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions choosing atleast TWO full questions from each part.***

PART – A

1 a. Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $-\pi \le x \le \pi$. **(07 Marks)**

b. Obtain a half range cosine for **(07 Marks)**

$$f(x) = \begin{cases} kx & \text{for } 0 \le x \le l/2 \\ k(l-x) & \text{for } l/2 \le x \le l. \end{cases}$$

c. Obtain the constant term and the coefficients of the first sine and cosine terms in the Fourier expansion of y as given in the following table. **(06 Marks)**

x :	0	1	2	3	4	5
Y :	9	18	24	28	26	20

2 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2 & \text{if } |x| \le 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$ and use it to evaluate $$\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos\frac{x}{2} dx.$$

(07 Marks)

b. Find the Fourier Cosine transform of e^{-x^2}. **(07 Marks)**

c. Using convolution theorem, find the inverse Fourier transform of $H(\alpha) = \frac{1}{(1+\alpha^2)^2}$. **(06 Marks)**

3 a. Form the partial differential equation by eliminating the arbitrary functions $F(x+2y) + G(x-3y) = 0$. **(07 Marks)**

b. Use the separation of variable technique to solve $3 U_x + 2 U_y = 0$. Given $U(x, 0) = 4 e^{-x}$. **(07 Marks)**

c. Solve $(x^2 - y^2 - z^2) p + 2xy q = 2xz$. **(06 Marks)**

4 a. Derive the one dimensional wave equation in the standard form. **(06 Marks)**

b. Obtain the various solutions of the Laplace's equation $Uxx + Uyy = 0$ by the method of separation of variables. **(07 Marks)**

c. A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set to vibrate by giving each point a velocity $V_0 \sin^3 \frac{\pi x}{l}$. Find the displacement $y(x, t)$. **(07 Marks)**

PART – B

5 a. Compute the real root of the equation $x \log_{10} x - 1.2 = 0$ correct to five decimal places using Regula Falsi method. **(07 Marks)**

b. Solve the following system of equations by Gauss – Seidel iteration method. $27x + 6y - z = 85$, $6x + 15y + 2z = 72$, $x + y + 54z = 110$. **(07 Marks)**

c. Find the largest Eigen value and the corresponding Eigen vector of the following matrix by using power method : $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$. Take $(1\ 0\ 0)^T$ as the initial Eigen vector. Carry out 4 iterations. **(06 Marks)**

6 a. Use Newton's divided difference formula to find f(8) given. **(07 Marks)**

x :	4	5	7	10	11	13
f(x) :	48	100	294	900	1210	2028

b. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.05 given **(07 Marks)**

x :	1	1.05	1.1	1.15	1.2	1.25	1.3
f(x) :	1	1.0247	1.04881	1.07238	1.09544	1.11803	1.14017

c. By Dividing the range into 6 equal parts, find the approximate value of $\int_0^{\pi} e^{\sin x}\, dx$ using simpsons 1/3rd rule. **(06 Marks)**

7 a. Derive Euler's equation in the form $\frac{d}{dx}\left(f - y'\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial x} = 0$. **(07 Marks)**

b. Find the extremal of the function $\int_0^{\pi/2} (y'^2 - y^2 + 4y\cos x)\,dx$ given $y(0) = 0$, $y(\pi/2) = 0$. **(06 Marks)**

c. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about the x – axis gives a minimum surface area. **(07 Marks)**

8 a. Find the Z – transform of i) n^2 ii) $\cos n\theta$. **(07 Marks)**

b. Find the inverse Z – transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. **(07 Marks)**

c. Solve the difference equation $Y_{n+2} + 2Y_{n+1} + Y_n = n$ with $Y_0 = Y_1 = 0$, using Z – transforms. **(06 Marks)**

USN

06CS34

Third Semester B.E. Degree Examination, Dec.08/Jan.09

Discrete Mathematical Structure

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions choosing at least two from each part.***

Part A

1 a. Define Tautology:
Show that $[(p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))] \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$ is tautology using laws. **(08 Marks)**

b. Define dual of logical statement. Write dual of logical statement, $(P \vee T_0) \wedge (q \vee F_0) \vee (r \wedge s \wedge T_0)$. **(04 Marks)**

c. Write truth values for NAND and NOR. **(02 Marks)**

d. Test validity of the following statement:

i) If there is strike by student, the examination will be postponed,
The Exam was not postponed
$\because$ There were no strike by student

ii) If Ravi studies, then he will pass DMS. If Ravi does not play cricket, then he will study
Ravi failed in DMS
$\therefore$ Ravi played cricket **(06 Marks)**

2 a. Define converse, inverse, contrapositive of implication, hence find converse, inverse, contrapositive for $\forall x, (x > 3) \rightarrow (x^2 > 9)$ where universal set is R. **(04 Marks)**

b. For any two odd integer mandn show that,
i) m + n is even. ii) mn is odd. **(06 Marks)**

c. Using quantifier method find whether following argument is valid,
If a triangle has two equal sides, then it is isoceles. If a triangle is isoceles, then it has two equal angles.
The triangle ABC does not have two equal angles
$\therefore$ ABC does not have two equal sides **(10 Marks)**

3 a. Find A and B if $A \cup B = \{1, 2, 4, 5, 7, 8, 9, 10\}$, $A \cap B = \{2, 4, 7\}$, $A - B = \{1, 8\}$. **(02 Marks)**

b. Using laws show that, $(A \cap B) \cup (A \cap B \cap \bar{C} \cap D) \cup (\bar{A} \cap B) = B$. **(06 Marks)**

c. 21 students took Maths exam having 3 questions and all of them answered at least one question of 5 fail to answer I[st] question, 6 fail to answer 2[nd] question 7 fail to answer 3[rd] question. If 9 answered all 3 question, find how many answered exactly one question. **(08 Marks)**

d. Find probability of two persons A and B contradicting when they narrate same story, given A speaks 60% true and B speaks 20% false. **(04 Marks)**

4 a. A sequence $\{a_n\}$ defined by $a_1 = 4$, $a_n = n + a_{n-1}$ for $n \geq 2$
Show that explicit expression of $a_n = 3 + \frac{1}{2}(n^2 + n)$. **(07 Marks)**

b. Prove by mathematical induction, $O[P(A)] = 2^n$. If O(A)=n where A is given set. **(07 Marks)**

c. Let R = {(1, 2), (1, 3), (2, 4), (3, 2)} be relation on A = {1, 2, 3, 4} find M(R) and $[M(R)]^2$ hence find R^2. **(06 Marks)**

Part B

a. Define equivalence relation and equivalence class with one example. **(07 Marks)**

b. For A = {a, b, c, d, x, y,z), define equivalence relation hence find equivalence class. Also find portion of A. **(07 Marks)**

c. Find $A \times B$, $A \times (B \cup C)$, $(A \cap B) \times C$ of A = {1, 2, 3, 4), B = {3, 4, 5, 6), C = {2, 4, 6). **(06 Marks)**

a. Define a function. Prove that function $f : A \to B$ is inversible. Iff it is one-one and onto. **(07 Marks)**

b. Define sterling number of 2^{nd} kind. If $|A| = 7$, $|B| = 4$ find number of onto function from A to B. Hence find S(7, 4). **(07 Marks)**

c. If $f(x) = x-1$, $g(x) = 3x$ $h(x) = \begin{cases} 0 & x \text{ even} \\ 1 & x \text{ odd} \end{cases}$. Show that $f \circ (g \circ n) = (f \circ g) \circ n$. **(06 Marks)**

a. Let G be set non-zero real. $a * b = \frac{ab}{2}$ for a, $b \in G$. Show that (G, *) is Abelian group. **(06 Marks)**

b. Define sub group. If H, K are subgroup of G. Prove that $H \cap K$ is also subgroup. Is $H \cup K$ is subgroup of G. Justify the answer. **(06 Marks)**

c. Define left and right cosets. State and prove Lagrange's theorem. **(08 Marks)**

a. The Generator matrix for an encoding function, $E : Z_2^3 \to Z_2^6$ is given by,

$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$. Find the code word assigned to 110 to 010. Also find associated parity check. **(08 Marks)**

b. Define: i) Hamming metric ii) The sphere of radius k centered at X. Give example in each case. **(06 Marks)**

c. Define ring with an example. **(06 Marks)**