MAT31

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NEW SCHEME

Third Semester B.E. Degree Examination, Dec.06 / Jan.07
Common to All Branches
Engineering Mathematics - III

Time: 3 hrs.] [Max. Marks:100

Note: *1. Answer any FIVE full questions choosing at least one from each part.*

Part - A

1 a. Using Regula-Falsi method compute the real root of the equation $x^3 - 4x - 9 = 0$ correct to two decimal places. **(06 Marks)**

b. The population of a town is given by the table :

Year	1951	1961	1971	1981	1991
Population in thousand	19.96	39.65	58.81	77.21	94.61

Using Newtons-Forward and Backward interpolation formula, calculate the increase in the population from the year 1955 to 1985. **(07 Marks)**

c. Construct an interpolating polynomial for the data given below using Newtons divided difference formula : **(07 Marks)**

x	2	4	5	6	8	10
$f(x)$	10	96	196	350	868	1746

2 a. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using Simpsons $\frac{3}{8}^{th}$ rule by considering seven ordinates and hence find the approximate value of π. **(06 Marks)**

b. Given that $\frac{dy}{dx} = x + y^2$ and $y(0) = 1$ find $y(0.2)$ in step size of 0.1 using modified Euler's method. Carry out two iterations in each step. **(07 Marks)**

c. From the data given below find y at $x = 1.4$ using Milne's predictor corrector formula. $\frac{dy}{dx} = x^2 + \frac{y}{2}$.

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514

(Use corrector formula twice) **(07 Marks)**

Part – B

3 a. An alternating current after passing through a half wave rectifier has the form,

$$i = \begin{cases} I_0 \sin\theta & \text{for } 0 \le \theta \le \pi \\ 0 & \text{for } \pi \le \theta \le 2\pi \end{cases}$$ (where I_0 in the maximum current).

Express i in a Fourier series. **(06 Marks)**

b. Obtain the Half-range cosine series for f(x) defined by,

$$f(x) = \begin{cases} K_x & \text{for } 0 \le x \le \frac{l}{2} \\ K(l-x) & \text{for } \frac{l}{2} \le x \le l \end{cases}$$ and hence find the sum of the series

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots\dots$ **(07 Marks)**

3 c. Express 'y' in a Fourier series up to second Harmonics given,

x^0	0	30	60	90	120	150	180	210	240	270	300	330	360
y	1.80	1.10	0.30	0.16	0.50	1.30	2.16	1.25	1.30	1.52	1.76	2.00	1.80

(07 Marks)

4 a. Find the Fourier transform of the function $f(x)=\begin{cases} a^2-x^2 & \text{for } |x|\le a \\ 0 & \text{for } |x|>a \end{cases}$. **(06 Marks)**

b. Find the Fourier sine transform of $e^{-|x|}$ and hence show that $\int_0^\infty \frac{x\sin mx}{1+x^2}dx=\frac{\pi e^{-m}}{2}$, $m>0$. **(07 Marks)**

c. Obtain the Z-transforms of $\cos n\theta$ and $\sin n\theta$. **(07 Marks)**

Part – C

5 a. Form the partial differential equation by eliminating arbitrary function from the equation $Z=y^2+2f\left(\frac{1}{x}+\log y\right)$. **(06 Marks)**

b. Solve $x^2(y-z)\frac{\partial z}{\partial x}+y^2(z-x)\frac{\partial z}{\partial y}=z^2(x-y)$. **(07 Marks)**

c. Solve $(p^2+q^2)y=qz$, using Charpits method. **(07 Marks)**

6 a. Under suitable assumptions derive one dimensional wave equation in the form $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$. **(08 Marks)**

b. Solve one dimensional heat equation $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$ under the following conditions.

i) $u(0,t)=u(4,t)=0,\ t>0$

ii) $u(x,0)=x(4-x)\ \ (0<x<4)$.

Using Explicit formula by taking $u=1$ and for $(0<t\le 1)$ **(12 Marks)**

Part – D

7 a. Test for consistency and solve $5x+3y+7z=4$, $3x+26y+2z=9$, $7x+2y+10z=5$. **(06 Marks)**

b. Apply Gause-Seidel iterative method to find the third approximate solution of the system of equations, $5x+2y+z=12$, $x+4y+2z=15$, $x+2y+5z=20$ with initial approximate solution (1, 0, 3). **(07 Marks)**

c. Use Rayleighs power method to find the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by taking initial eigen vector as $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(Carry out 4 iterations). **(07 Marks)**

8 a. With usual notation derive Euler's equation in the form $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$. **(06 Marks)**

b. Find the curve passing through the points (x_1y_1) and (x_2y_2) which when rotated about x-axis gives a minimum surface area. **(07 Marks)**

c. Find the external of the functional,

$$\int_{x_0}^{x_1}(x+y')y'dx.$$ **(07 Marks)**

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NEW SCHEME

Third Semester B.E. Degree Examination, July 2007
CS / IS
Discrete Mathematical Structures

Time: 3 hrs.] [Max. Marks:100

Note : ***Answer any FIVE full questions.***

1 a. For any three sets A, B, C prove that
$(A - B) - C = A - (B \cup C) = (A - C) - (B - C)$ **(07 Marks)**

b. A professor has two dozen introductory textbooks on computer science and is concerned about their coverage of the topics (A) compiler, (b) data structures and (C) operating systems. The following data are the numbers of books that contain material on these topics :
$|A| = 8,\ |B| = 13,\ |C| = 13,\ |A \cap B| = 5,\ |A \cap C| = 3,\ |B \cap C| = 6,\ |A \cap B \cap C| = 2$
i) How many of the textbooks include material on exactly one of these topics?
ii) How many do not deal with any of the topics?
iii) How many have no material on compilers? **(07 Marks)**

c. For all $n \in Z^{+}$ show that if $n \geq 24$, then n can be written as a sum of 5's and or 7's. **(06 Marks)**

2 a. Define Tautology. Prove that, for any propositions p, q, r, the compound proposition $\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is a tautology. **(05 Marks)**

b. Prove the following logical equivalences without using truth tables :
i) $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$
ii) $[(\neg p \vee \neg q) \to (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$ **(05 Marks)**

c. For any statements p, q prove that
i) $\neg(p \downarrow q) \Leftrightarrow (\neg p \uparrow \neg q)$
ii) $\neg(p \uparrow q) \Leftrightarrow (\neg p \downarrow \neg q)$ **(05 Marks)**

d. Test the validity of the following argument :
I will become famous or I will not become a musician
I will become a musician. Therefore, I will become famous. **(05 Marks)**

3 a. Write down the following propositions in symbolic form and find its negation.
i) If all triangles are right angled, then no triangle is equiangular.
ii) For all integers n, if n is not divisible by 2, then n is odd. **(07 Marks)**

b. Prove that the following argument is valid :

$$\begin{array}{l} \forall x[p(x) \to q(x)] \\ \forall x[q(x) \to r(x)] \\ \hline \therefore\ \forall x[p(x) \to r(x)] \end{array}$$

where p(x), q(x) and r(x) are open statements that are defined for a given universe. **(07 Marks)**

c. Provide a proof by contradiction for the following :
For every integer n, if n^2 is odd, then n is odd. **(06 Marks)**

Contd.... 2

4 a. Define Cartesian product of two sets. For A, B, C $\subseteq$ U, prove that

$$A \times (B - C) = (A \times B) - (A \times C)$$ **(05 Marks)**

b. Let A = {1, 2, 3, 4} an R be a relation on A defined by aRb if and only i "a divides b". Write down the relation matrix M(R) and draw its digraph. **(05 Marks)**

c. For A = {1, 2, 3, 4}, let R and S be the relations on A defined by R = {(1, 2) (1, 3) (2, 4) (4, 4)} and S = {(1, 1) (1, 2) (1,3) (2, 3) (2, 4)}. Find R_oS, S_oR, R^2, S^2 and R^3 **(05 Marks)**

d. Let A = {a, b, c, d, e}. Consider the partition P = {{a, b}, {c, d}, {e}} of A. Find the equivalence relation inducing this partition. **(05 Marks)**

5 a. If R is an equivalence relation on a set A, and x, y $\in$ A, then prove that
   i) $x \in [x]$
   ii) x R y if and only if $[x] = [y]$
   iii) $[x] = [y]$ or $[x] \cap [y] = \phi$ **(07 Marks)**

b. Define a partially ordered set. The directed graph for a relation R on a set A = {1, 2, 3, 4} is as shown below

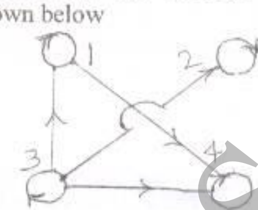

Fig.5(b)

   i) Verify that (A, R) is a poset and find its Hasse diagram.
   ii) Topologically sort (A, R) **(07 Marks)**

c. Let (A, R) be a poset with B $\subseteq$ A. Define
   i) Lower bound of B
   ii) Upper bound of B
   iii) Greatest lower bound of B
   iv) Least upper bound of B. **(06 Marks)**

6 a. Define
   i) One-to-one function
   ii) On to function

Give one example each.

Let f, g, h be functions from Z to Z defined by

$$f(x) = x - 1, \quad g(x) = 3x, \quad h(x) = \begin{cases} 0, & \text{if } x \text{ is even} \\ 1, & \text{if } x \text{ is odd} \end{cases}$$

Verify that $f_o(g_oh) = (f_og)_oh$. **(07 Marks)**

b. Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of the set A = {1, 2, 3, 4, 5, 6}.
   i) Write p as a product of disjoint cycles.
   ii) Compute p^{-1}
   iii) Compute p^2 and p^3
   iv) Find the smallest integer R such that $p^R = I_A$ **(07 Marks)**

c. Define characteristic function. For any sets A, B $\subseteq$ U, prove that :
   i) $f_{\overline{A}}(x) = 1 - f_A(x)$
   ii) $f_{A \cap B}(x) = f_A(x) . f_B(x)$ **(06 Marks)**

7 a. Define Abelian group. Prove that a group G is abelian if and only if for all a, b $\in$ G, $(a, b)^{-1} = a^{-1} b^{-1}$ **(06 Marks)**
b. Define Subgroup. If H, K are subgroups of a group G, prove that H $\cap$ K is also a subgroup of G. Is H $\cup$ K a subgroup of G? Justify your answer. **(06 Marks)**
c. Define left and right cosets. State and prove Lagrange's theorem. **(08 Marks)**

8 a. Define a ring. In a ring (R, +, $\bullet$), for all a, b $\in$R prove that
   i) $a . 0 = 0 . a = 0$
   ii) $a(-b) = (-a)b = -(ab)$ **(07 Marks)**
b. Define group Homomorphism. Let f be a homomorphism from a group G_1 to a group G_2. Prove that
   i) If e_1 is the identity in G_1 and e_2 is the identity in G_2 then $f(e_1) = e_2$.
   ii) $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G_1$ **(07 Marks)**
c. Define a group code. Consider the encoding function $E : Z_2^2 \to Z_2^6$ of the triple repetition code where E(00) = 000000, E(10) = 101010, E(01) = 010101 E(11) = 111111. Prove that $C = E(Z_2^2)$ is a group code. **(06 Marks)**