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06MAT41

Fourth Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting at least TWO questions from each part.***
2. Use of statistical tables is permitted.

PART – A

1 a. Find the y(0.1) correct to 6 decimal places by Taylor series method when $dy/dx = xy + 1$, $y(0) = 1.0$. (Consider upto 4th degree term). **(06 Marks)**

b. Using Runge-Kutta method of order 4, compute y(0.2) for the equation, $y' = y - \frac{2x}{y}$, $y(0) = 1.0$ (Take $h = 0.2$). **(07 Marks)**

c. Given that $y' = x^2(1+y)$ and $y(1) = 1.0$, $y(1.1) = 1.233$, $y(1.2) = 1.548$ and $y(1.3) = 1.979$, compute y(1.4) by Adams-Bashforth method. Apply correct formula twice. **(07 Marks)**

2 a. Show that Z^n is analytic. Hence find its derivative. **(06 Marks)**

b. Find a bilinear transformation which maps the points 0, 1, i in the Z-plane onto $1 + i$, $-i$, $2 - i$ in the W plane. **(07 Marks)**

c. Find the analytic function u + iv, where u is given to be $u = e^x[(x^2 - y^2)\cos y - 2xy \sin y]$. **(07 Marks)**

3 a. Derive Couchy's integral formula in the form

$$f(a) = \frac{1}{2\pi i}\int_c \frac{f(z)dz}{z-a}$$ **(06 Marks)**

b. Expand $f(z) = \frac{7z^2 + 9z - 18}{z^3 - 9z}$ in the Laurent series that is valid for
i) $|z| > 3$ ii) $0 < |z - 3| < 3$. **(07 Marks)**

c. Evaluate $\int_c \tan z\, dz$, where c is $|z| = 2.5$ **(07 Marks)**

4 a. Find the series solution of $\frac{d^2y}{dx^2} + xy = 0$. **(06 Marks)**

b. Express $x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials. **(07 Marks)**

c. Reduce the differential equation $x\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} + k^2xy = 0$ to Bessel's equation. Obtain the solution. **(07 Marks)**

PART – B

5 a. Fit a curve of the form $y = ab^x$ for the data given below: **(06 Marks)**

x :	2	4	6	8	10	12
y :	1.8	1.5	1.4	1.1	1.1	0.9

b. Find the coefficient of correlation for the following data: **(07 Marks)**

x :	55	56	58	59	60	60	62
y :	35	38	39	38	44	43	45

c. In a certain college 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body.

i) What is the probability that mathematics is being studied?

ii) If a student is selected a random and is found to be studying mathematics, find the probability that the student is a girl. **(07 Marks)**

6 a. Suppose a random variable X takes the values –3, –1, 2 and 5 with respective probabilities $\frac{2k-3}{10}, \frac{k-2}{10}, \frac{k-1}{10}, \frac{k+1}{10}$. Find the value of k and i) find $P[-3 < X < 4]$ and ii) $P[X \le 2]$. **(06 Marks)**

b. Suppose that the student IQ scores form a normal distribution with mean 100 and standard deviation 20. Find the percentage of students whose i) score is less than 80 ii) score falls between 90 and 140, iii) Score more than 120. **(07 Marks)**

c. Obtain mean and variance of binomial distribution function. **(07 Marks)**

7 a. A sample of 1000 days is taken from meteorological records of a certain district and 120 of them are found to be foggy. What are the probable 99% confidence limits to the proportion of foggy days in the district? **(06 Marks)**

b. The following table gives the number of bus accidents that occurred during the various days of the week. Find whether the accidents are uniformly distributed over the week, using χ^2 test. **(07 Marks)**

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of accidents	14	16	8	12	11	9	14	84

c. The life X of certain computer is approximately normally distributed with mean 800 hours and standard deviation 40 hours. If a random sample of 30 computers has an average life of 788 hours, test the hypothesis that $\mu = 800$ hours against the alternate hypothesis $\mu \neq 800$ hours at i) 0.5% and 1% level of significance. **(07 Marks)**

8 a. A fair coin is tossed 4 times. Let X denote the number of heads occurring and let Y denote the longest string of heads occurring. Find the joint distribution function of X and Y. **(06 Marks)**

b. A man's gambling luck follows a pattern. If he wins a game the probability of winning the next game is 0.6. However, if he loses a game, the probability of losing the next game is 0.7. There is an even chance that he wins the first game.

i) Find the transition matrix of the Markov process. ii) Find the probability that he wins the third game. iii) Find out how often, in the long run, he wins. **(07 Marks)**

c. Explain: i) Transient state ii) Absorbing state and iii) Recurrent state by means of an example each. **(07 Marks)**

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Fourth Semester B.E. Degree Examination, December 2010

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting at least TWO questions from each part.***
2. Any missing data may be suitably assumed.

PART – A

1 a. Given $\frac{dy}{dx} + y - x^2 = 0$, $y(0) = 1$, $y(0.1) = 0.9052$, $y(0.2) = 0.8213$. Find correct to four decimal places y(0.3) and y(0.4) using modified Euler's method. **(07 Marks)**

b. Apply Runge – Kutta method of order four, to compute y(2.0). Given $10\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$, taking $h = 0.1$. **(07 Marks)**

c. The following table gives the solution of $\frac{dy}{dx} = x - y^2$. Find the value of y at x = 0.8, using Milne's predictor and corrector formulae.

X	0	0.2	0.4	0.6
Y	0	0.02	0.07	0.17

(06 Marks)

2 a. Show that polar forms of Cauchy's Riemann equation are
$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial V}{\partial \theta}, \frac{\partial V}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$. Deduce that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. **(07 Marks)**

b. Determine the analytic function $w = u + i\upsilon$ if $V = \log(x^2 + y^2) + x - 2y$. **(07 Marks)**

c. Find the Bilinear transformation which maps the points $z = 1, i, -1$ into $w = 0, 1, \infty$. **(06 Marks)**

3 a. State and prove Cauchy's integral formula. **(07 Marks)**

b. Find the Laurent series of $\frac{3x^2 - 6z + 2}{z^3 - 3z^2 + 2z}$. i) $1 < |z| < 2$ ii) $|z| > 2$. **(07 Marks)**

c. Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where c is $|z| = 3$ using Cauchy's residue theorem. **(06 Marks)**

4 a. Solve the equation in series $\frac{d^2y}{dx^2} + x^2 y = 0$. **(07 Marks)**

b. Obtain the series solution of Bessel's differential equation in the form $y = AJ_n(x) + BJ_{-n}(x)$. **(07 Marks)**

c. If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$, find the value of a, b, c, d. **(06 Marks)**

PART – B

5 a. Fit a curve of form $y = ab^x$ and hence estimate y when x = 8.

X	1	2	3	4	5	6	7
Y	87	97	113	129	202	195	193

(07 Marks)

b. If θ is the angle between the lines of regression then show that

$$\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$$ (07 Marks)

c. State and prove Baye's theorem. (06 Marks)

6 a. The pdf of a variate X is given by the following table :

X	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

For what value of k, this represents a valid probability distribution?
Also find : i) $P(x \geq 5)$ ii) $P(3 < x \leq 6)$. (07 Marks)

b. Given that 2% of the fuses manufactured by a firm are defective, find by using Poisson distribution, the probability that a box containing 200 fuses has
i) No defective fuses ii) 3 or more defective fuses iii) At least one defective fuse. (07 Marks)

c. The marks of 100 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be i) less than 65 ii) more than 75 iii) between 65 and 75. (06 Marks)

7 a. Explain the following terms :
i) Null hypothesis ii) Type I and type II error iii) Confidence limits. (07 Marks)

b. A sample of 100 days is taken from a coastal town of a certain district and of 10 of them are found to be very hot. What are the probable limits of the percentage of hot days in the district? (07 Marks)

c. A certain stimulus administrated to each of the 12 patients resulted in the following change in blood pressure.
5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure? ($t_{0.05}$ for 11 df = 2.201). (06 Marks)

8 a. The joint probability distribution of two random variables x and y is as follows :

x \ y	-2	-1	4	6
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine :
i) The marginal distribution of x and y ii) Co variance of x and y iii) Correlation of x and y. (07 Marks)

b. Verify that the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 \end{bmatrix}.$$

is a regular stochastic matrix. (07 Marks)

c. Explain:
i) Absorbing state of Markov chain ii) Transient state iii) Recurrent state. (06 Marks)

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