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**Third Semester B.E. Degree Examination, December 2011**  
**Discrete Mathematical Structures**

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions**  
**atleast TWO questions from each part.**

**PART – A**

- 1 a. If  $N$  is the set of positive integers and  $R$  is the set of real numbers, examine which of the following sets is empty :
- $\{x \mid x \in N, 2x + 7 = 3\}$
  - $\{x \mid x \in R, x^2 + 4 = 6\}$
  - $\{x \mid x \in R, x^2 + 3x + 3 = 0\}$ . (04 Marks)
- b. Using Venn diagrams, investigate the truth or falsity of :
- $A \Delta (B \cap C) = (A \Delta B) \cap (A \Delta C)$
  - $A - (B \cup C) = (A - B) \cap (A - C)$  for any three sets  $A, B, C$ . (06 Marks)
- c. Simplify the following :
- $A \cap (B - A)$
  - $(A \cap B) \cup (A \cap \bar{B}) \cup (A \cap B \cap \bar{C} \cap D)$ . (05 Marks)
- d. A fair coin is tossed five times. What is the probability that the number of heads always exceeds the number of tails as each outcome is observed. (05 Marks)
- 2 a. Write the following in symbolic form and establish if the argument is valid : If  $A$  gets the supervisor's position and works hard, then he will get a raise. If he gets a raise, then he will buy a new car. He has not bought a new car. Therefore  $A$  did not get the supervisor's position or he did not work hard. (05 Marks)
- b. Verify the following without, using truth tables :
- $$[(p \rightarrow q) \wedge (\neg r \vee s) \wedge (p \vee r)] \therefore \neg q \rightarrow s. \quad (05 \text{ Marks})$$
- c. Define tautology. Show that  $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$  is a tautology, by constructing a truth table. (05 Marks)
- d. Show that the following argument is invalid by giving a counter example :
- $$[(p \wedge \neg q) \wedge \{p \rightarrow (q \rightarrow r)\}] \rightarrow \neg r. \quad (05 \text{ Marks})$$
- 3 a. Verify if the following is valid :
- $$\forall x [p(x) \vee q(x)] ; \exists x \neg p(x)$$
- $$\forall x [\neg g(x) \vee r(x)]$$
- $$\forall x [s(x) \rightarrow \neg r(x)] \therefore \exists x \neg s(x). \quad (05 \text{ Marks})$$
- b. Prove that for all real numbers  $x$  and  $y$ , if  $x + y > 100$ , then  $x > 50$  or  $y > 50$ . (05 Marks)
- c. Determine if the argument is valid or not. All people concerned about the environment, recycle their plastic containers.  $B$  is not concerned about the environment. Therefore,  $B$  does not recycle his plastic containers. (05 Marks)
- d. Negate and simplify :
- $\forall x [p(x) \wedge \neg q(x)]$
  - $\exists x [(p(x) \vee q(x)) \rightarrow r(x)]$ . (05 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- 4 a. Define the following : i) Well ordering principle  
ii) Principle of mathematical induction. (04 Marks)
- b. Establish the following by mathematical induction :  

$$\sum_{i=1}^n i(2^i) = 2 + (n-1)2^{n+1}. \quad (05 \text{ Marks})$$
- c. Find a unique solution for the recurrence relation :  $4a_n - 5a_{n-1} = 0, n \geq 1, a_0 = 1. \quad (05 \text{ Marks})$
- d. Let  $F_n$  denote the  $n^{\text{th}}$  Fibonacci number :  
 Prove that 
$$\sum_{i=1}^n \frac{F_{(i-1)}}{2^i} = 1 - \frac{F_{(n+2)}}{2^n}. \quad (06 \text{ Marks})$$

## PART - B

- 5 a. Define Cartesian product of two sets. For non-empty sets A, B, C prove that,  
 $A \times (B \cap C) = (A \times B) \cap (A \times C). \quad (04 \text{ Marks})$
- b. For each of the following functions, determine whether it is 1-1 :  
 i)  $f: Z \rightarrow Z, f(x) = 2x + 1$       ii)  $f: Z \rightarrow Z, f(x) = x^3 - x. \quad (06 \text{ Marks})$
- c. Let  $A = B = C = R, f: A \rightarrow B, f(a) = 2a + 1; g: B \rightarrow C, g(b) = b/2$ . Compute gof and show that it is invertible. (05 Marks)
- d. Let  $\Delta ABC$  be an equilateral triangle of side 1. Show that if we select 10 points in the interior, there must be at least two points whose distance apart is less than  $1/3$ . (05 Marks)
- 6 a. For each of the following relations, determine if the relation R is reflexive, symmetric, antisymmetric or transitive :  
 i) On the set of all lines in the plane,  $l_1 R l_2$  if line  $l_1$  is perpendicular to line  $l_2$  (05 Marks)  
 ii) On Z,  $x R y$  if  $x - y$  is even. (05 Marks)
- b. For  $A = \{1, 2, 3, 4\}$ , let  $R = \{(1, 1), (1, 2), (2, 3), (3, 3), (3, 4)\}$  be a relation on A. Draw the digraph of  $R^2$  and find the matrix  $M(R^2)$ . (05 Marks)
- c. Draw the Hasse diagram for all the positive integer division of 72. (05 Marks)
- d. Let  $A = \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}$ . Define R on A by  $(x_1, y_1) R (x_2, y_2)$  if  $x_1 y_1 = x_2 y_2$ . Verify that R is an equivalence relation on A. (05 Marks)
- 7 a. For a group  $(G_1, \cdot)$ , prove that it is abelian if  $(a, b)^2 = a^2 b^2$  for all  $a, b \in G$ . (05 Marks)
- b. Let  $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ . Verify that  $(A, A^2, A^3, A^4)$  form an abelian group under matrix multiplication. (06 Marks)
- c. Define a cyclic group. Verify that  $(Z_5, \cdot)$  is cyclic. Find a generator of this group. Examine if it has any subgroups. (09 Marks)
- 8 a. Determine whether  $(Z, \oplus, \otimes)$  is a ring with the binary operations  $x \oplus y = x + y - 7$ ,  
 $x \otimes y = x + y - 3xy$  for all  $x, y \in Z$ . (06 Marks)
- b. The  $(5m, m)$  five times repetition code has encoding function  $E: Z_2^m \rightarrow Z_2^{5m}$ . Decoding with  $D: Z_2^{5m} \rightarrow Z_2^m$  is done by majority rule. With  $p = 0.05$ , what is the probability for the transmission and correct decoding of the signal 110. (06 Marks)
- c. What is the minimum distance of a code consisting of the following code words :  
 001010, 011100, 010111, 011110, 101001? What kind of errors can be detected? (03 Marks)
- d. The encoding function  $E: Z_2^2 \rightarrow Z_2^5$  is given by the generator matrix  $G = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$ .  
 What is the error detection capacity of the code? (05 Marks)

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